Lecture-Ten

Boundary Layer Equations

1- Von Karman Integral Equation.

From the 2nd law of Newton's and momentum equation for a C.V as shown in Fig.(1) the following equations can be written

$\sum F = \frac{d}{dt}\int_{c.v} \rho \vec{V} d\forall + \int_{c.s} \rho \vec{V}(\vec{V}.n)dA.$

$\sum F_x = \rho\left(u + \frac{\partial u}{\partial x}dx\right)^2 dy - \rho u^2 dy + \rho\left(v + \frac{\partial v}{\partial y}dy\right)\left(u + \frac{\partial u}{\partial y}\,dy\right)dx - \rho u\, v dx$

$\sum F_x = \rho\left[u^2 + 2u\frac{\partial u}{\partial x}\,dx + \left(\frac{\partial u}{\partial x}\,dx\right)^2\right]dy - \rho u^2 dy + \rho v u dx + \rho v\frac{\partial u}{\partial y}\,dy dx + \rho\, u\frac{\partial v}{\partial y}dy\,dx +$
$\rho\frac{\partial v}{\partial y}.\frac{\partial u}{\partial y}\,(dy)^2 * dx - \rho u v\,dx$

$\sum F_x = \rho\left(2\,u\frac{\partial u}{\partial x} + \,u\frac{\partial v}{\partial y} + \,v\frac{\partial u}{\partial y}\right)dy\,dx$

After disregarding second – order terms & $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$ from C.E.

$\therefore\ \sum F_x = \,\rho\left(\,u\frac{\partial u}{\partial x} + \,v\frac{\partial u}{\partial y}\right)dxdy$ (1)

The summation of forces on C.V may be written as

$\sum F_x = \,p\,dy - \left(p + \frac{\partial p}{\partial x}\,dx\right)dy + \left(\tau_x + \frac{\partial \tau_x}{\partial y}\,dy\right)dx - \tau_x dx$

$\sum F_x = \,p\,dy - pdy - \frac{\partial p}{\partial x}dxdy + \tau_x dy + \frac{\partial \tau_x}{\partial y}dydx - \tau_x dx$

$\sum F_x = \left(-\frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial y}\right)dxdy$ (2)

Equating Eq's (1 & 2)

$\rho\left(u\frac{\partial u}{\partial x} + \,v\frac{\partial u}{\partial y}\right)dx\,dy = \left(-\frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial y}\right)dxdy$ (3)

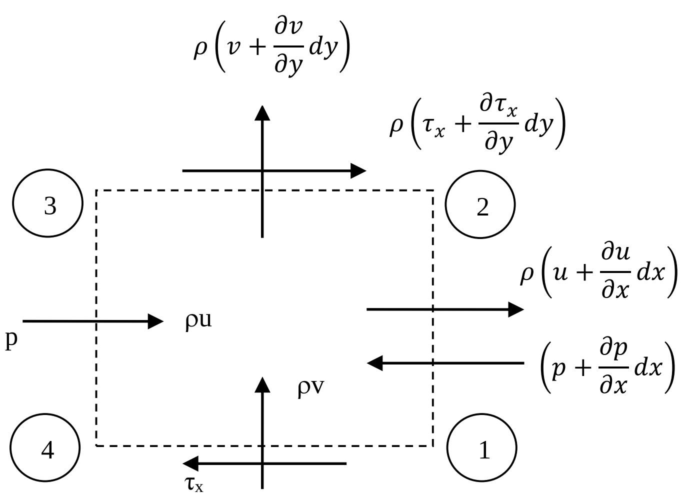

Figure 1: Distribution of pressure forces on control volume.

The shear stress is very nearly equal to

$\tau_x = \mu \frac{\partial u}{\partial y}$; Substituting in Eq.(3) gives

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(\frac{\mu}{\rho}\right)\frac{\partial^2 u}{\partial y^2} \qquad (4)$$

Table 1: Masses and momentum fluxes on control volume faces

Surface	Mass flux	Flux of x-momentum
(1-2)	$\rho\left(u + \frac{\partial u}{\partial x}\, dx\right) dy$	$\rho\left(u + \frac{\partial u}{\partial x} dx\right)^2 dy$
(3-4)	$\rho u\, dy$	$\rho u^2 dy$
(2-3)	$\rho\left(v + \frac{\partial v}{\partial y}\, dy\right) dx$	$\rho\left(v + \frac{\partial v}{\partial y}\, dy\right)\left(u\frac{\partial u}{\partial y}\, dy\right) dx$
(4-1)	$\rho v\, dx$	$\rho v\, u dx$

From the Integral method of momentum equation for Von Karman integral as follows

$$\int_0^h \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) dy = \int_0^h -\frac{1}{\rho}\frac{dp}{dx}\, dy + \int_0^h v\frac{\partial^2 u}{\partial y^2}\, dy \qquad (5)$$

Where h is an undefined distance from the wall to outside the boundary layer. Integrating the second term in the integral by parts, we have

$$[\int_0^h v\frac{\partial u}{\partial y} dy] = [vu]_0^h - \int_0^h u\frac{\partial v}{\partial y} dy ----(a)$$

From C.E, v at y=h is given by ;

$$v = \int_0^h \frac{\partial v}{\partial y} dy = -\int_0^h \frac{\partial u}{\partial x} dy \qquad ----(b)$$

Substitute Eq. (b in a) and $u = U_\infty\ at\ y = h$

$$[\int_0^h v\ \frac{\partial u}{\partial y}\ dy] = -U_\infty \int_0^h \frac{\partial u}{\partial x}\ dy + \int_0^h u\frac{\partial u}{\partial x}\ dy \quad ----(c)$$

<u>CASE (A)</u> $\frac{\partial p}{\partial x} = 0$

Substitute Eq. (c) in Eq's (8.9& 8.10) and neglecting $\frac{\partial p}{\partial x} = 0$

$$\int_0^h \rho\left(u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x} - U_\infty\frac{\partial u}{\partial x}\right) dy = -\tau_0$$

$$\int_0^h \rho\ \left(2u\frac{\partial u}{\partial x} - U_\infty\frac{\partial u}{\partial x}\right) dy = -\tau_0 \qquad ------(e)$$

$$\int_0^h \rho\ \left(U_\infty\frac{\partial u}{\partial x} - 2u\frac{\partial u}{\partial x}\right) dy = \tau_0 \qquad ------(f)$$

Since $\frac{\partial u^2}{\partial x} = 2u\frac{\partial u}{\partial x}$

$$\int_0^h \rho\ \left(\left(U_\infty\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial x}(u^2)\right) dy = \tau_0$$

$$\tau_0 = \rho\frac{d}{dx}\int_0^h u\ (U_\infty - u) dy \qquad since\ \ h = \delta$$

$$\tau_0 = \rho\frac{d}{dx}\int_0^\delta u\ (U_\infty - u) dy$$

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy \qquad (6)$$

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}$ (7)

Since $\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$ (8)

Eq. (8) is the Von Karman equation without pressure gradient dp/dx=0

CASE (B) $\frac{\partial p}{\partial x} \neq 0$

When the pressure gradient in Eq's (4 or 5) is included and from adding it in Eq. (e) as followes

$-\frac{1}{\rho}\frac{dp}{dx} = U_\infty \frac{dU_\infty}{dx}\ from\ Bournalli's\ Equation$

Now $\int_0^h \rho\left(-U_\infty \frac{\partial u}{\partial x} + 2u\frac{\partial u}{\partial x} - U_\infty \frac{dU_\infty}{dx}\right) dy = -\tau_0$ (9)

$\frac{\partial}{\partial x}(u\, U_\infty) = U_\infty \frac{\partial u}{\partial x} + u\frac{\partial U_\infty}{\partial x}$

$U_\infty \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(u\, U_\infty) - u\frac{\partial U_\infty}{\partial x}$ Substituted in Eq. (9) after multiply by (-1)

$\int_0^h \left[2u\frac{\partial u}{\partial x} - U_\infty \frac{\partial U_\infty}{\partial x} + u\frac{\partial U_\infty}{\partial x} - \frac{\partial}{\partial x}(u\, U_\infty)\right] dy = -\frac{\tau_0}{\rho}$

In above eqn. $2u\frac{\partial u}{\partial x} = \frac{\partial u^2}{\partial x}$

$\int_0^h \frac{\partial}{\partial x}[u(u - U_\infty]\, dy - \frac{dU_\infty}{dx}\int_0^h (U_\infty - u)dy = -\frac{\tau_0}{\rho}$

$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$

$\delta_d = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$

$\therefore \frac{\tau_0}{\rho} = \frac{d}{dx}(U_\infty^2\theta) + U_\infty\, \delta_d \frac{dU_\infty}{dx}$

$\frac{\tau_0}{\rho} = U_\infty^2 \frac{d\theta}{dx} + \theta * 2U_\infty \frac{dU_\infty}{dx} + U_\infty\, \delta_d \frac{dU_\infty}{dx}$

$\frac{\tau_0}{\rho} = U_\infty^2 \frac{d\theta}{dx} + (2\theta + \delta_d)U_\infty \frac{dU_\infty}{dx}$ (10)

We assume

$C_f = \frac{\tau_0}{\frac{1}{2}\rho\, U_\infty^2}\ and\ H = \frac{\delta_d}{\theta}$

C_f is the friction factor and H is the shape factor.

$\therefore \frac{C_f}{2} = \frac{d\theta}{dx} + \theta(2 + H)\ \frac{1}{U_\infty}\frac{dU_\infty}{dx}$ (11)

From Eq's (10 and 11) the case of $\frac{dp}{dx} = 0$ & $\frac{dU_\infty}{dx} = 0$

No gradient of pressure along the x-axis

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx} = \frac{1}{2}\, cf$ (12)

2- Approximate Solution to the Laminar B.L.

We have four conditions that proposed velocity profile should satisfy on flat plat with zero pressure gradients.

$u = 0\ at\ y = 0$

$u = U_\infty\ at\ y = \delta$

$\frac{\partial u}{\partial x} = 0\ at\ y = \delta$

$\frac{\partial^2 u}{\partial y^2} = 0 \; at \; y = 0$

Let $\frac{u}{U_\infty} = A + By + Cy^2 + Dy^3$ is a cubic polynomial will satisfy the four conditions.

From above conditions

At y=0 , u=0

$\frac{u}{U_\infty} = 0 = A \qquad ------(a)$

At $y = \delta \qquad u = U_\infty$

$\frac{U}{U_\infty} = 1 = B\delta + C\delta^2 + D\delta^3 \qquad ------(b)$

At $y = \delta \qquad \frac{\partial u}{\partial y} = 0$

$\frac{\partial u}{\partial y} = B + 2cy + 3Dy^2$

$B + 2C\delta + 3D\delta^2 = 0 \qquad ------(c)$

At y=0

$\frac{\partial^2 y}{\partial y^2} = 0 = 2C + 6Dy ---\rightarrow C = 0 \qquad ----(d)$

From Eq. (c) → $B = -3D\delta^2 \qquad ------(e)$

From Eq. (b) $B = \frac{1}{\delta} - D\delta^2 \qquad ----(f)$

Equating (*e* & *f*)

$-3D\delta^2 = \frac{1}{\delta} - D\delta^2 \qquad ----\rightarrow D = -\frac{1}{2\delta^3} \; ; \quad B = \frac{3}{2\delta}$

Hence a good approximation for the velocity profile in a laminar flow is

$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

Let us now use this velocity profile to find $\delta(x) and \; \tau_0(x)$. From Von Karman's integral Eq's (6 & 7)

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$

Assume the velocity take the following form $\frac{u}{U_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$ in the B.L. Calculate the thickness and the wall shear.

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta \left(\frac{3}{2}\frac{y}{\delta} - \frac{y^3}{2\delta^3}\right)\left(1 - \frac{3y}{2\delta} + \frac{y^3}{2\delta^3}\right) dy$

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta (\frac{3}{2}\frac{y}{\delta} - \frac{9y^2}{4\delta^2} + \frac{3y^4}{4\delta^4} - \frac{y^3}{2\delta^3} + \frac{3y^4}{4\delta^4} - \frac{y^6}{4\delta^6})\, dy$

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta \left(\frac{3}{2}\frac{y}{\delta} - \frac{9y^2}{4\delta^2} + \frac{6y^4}{4\delta^4} - \frac{y^3}{2\delta^3} - \frac{y^6}{4\delta^6}\right) dy$

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\left[\frac{3y^2}{4\delta} - \frac{9y^3}{12\delta^2} + \frac{6y^5}{20\delta^4} - \frac{y^4}{8\delta^3} - \frac{y^7}{28\delta^6}\right]_0^\delta$

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\left[\frac{3}{4}\delta - \frac{9}{12}\delta + \frac{6}{20}\delta - \frac{\delta}{8} - \frac{\delta}{28}\right]$

$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\delta}{dx}(0.1392) = 0.1392\,\frac{d\delta}{dx}$

$\therefore \; \tau_0 = 0.1392\,\rho U_\infty^2\, d\delta/dx \qquad (13)$

At the wall we know that $\tau_0 = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$ or using the cubic profile

$\frac{\partial u}{\partial y}\Big|_{y=0} = B + 2Cy + 3Dy^2 = B = (\frac{3}{2\delta})U_\infty$

$$\therefore\ \tau_0 = \mu\left(U_\infty \frac{3}{2\delta}\right) \quad (14)$$

Equating the foregoing expression (13 & 14) for $\tau_0(x)$, we find that

$0.139\rho U_\infty^2 \frac{d\delta}{dx} = \mu\left(U_\infty \frac{3}{2\delta}\right)$

$$\delta\, d\delta = \frac{\frac{3}{2}\mu}{0.139\rho U_\infty} dx = 10.8 \frac{\nu}{U_\infty} dx \quad (15)$$

From using at $\delta = 0\ at\ x = 0$ (the leading edge) Eq. (15) is integrated to give

$\delta = 4.65\sqrt{\nu x/U_\infty} \quad multiply\ by \frac{\sqrt{x}}{\sqrt{x}}$

$$\delta = 4.65 \frac{x}{\sqrt{Re_x}} \quad (16)$$

Where Re_x is the local Reynolds number. Substituted Eq. (16) in Eq.(14) giving the wall shear as

$\tau_0 = 0.323\ \rho U_\infty^2 \sqrt{\frac{\nu}{x\, U_\infty}}$

$$\tau_0 = \frac{0.323\rho U_\infty^2}{\sqrt{Re_x}} \quad (17)$$

The shearing stress is made dimensionless by dividing by $\frac{1}{2}\ \rho U_\infty^2$. The local skin friction coefficient C_f its

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2} = \frac{0.646}{\sqrt{U_\infty \frac{x}{\nu}}} = \frac{0.646}{\sqrt{Re_x}} \quad (18)$$

If the wall shear is integrated over the length L, the result per unit width is the drag force.

$F_D = \int_0^L \tau_0 dx = 0.646\ \rho U_\infty \sqrt{U_\infty\ L\nu}$ Where τ_0 from Eq. (17)

$$F_D = \frac{0.646\ \rho U_\infty^2 L}{\sqrt{Re_L}} \quad (19)$$

Or $F_D = \int_0^L \tau_0\, dx = \tau_0 . L = C_F . \frac{1}{2}\rho U_\infty^2 . L$ since τ_0 from Eq. (18)

$$\therefore\ C_F = \frac{F_D}{\frac{1}{2}\rho U_\infty^2 . L} = \frac{0.646\ \rho U_\infty^2 . L}{\frac{1}{2}\rho\ U_\infty^2 . L\sqrt{Re_L}} = \frac{1.292}{\sqrt{Re_L}} \quad (20)$$

Where Re_L is the Reynolds number at the end of flat plate.

<u>Ex.1</u>

Assume a parabolic velocity profile and calculate the B.L thickness and the wall shear. Compare with those calculate above.

<u>Sol.</u>

The parabolic velocity profile is assumed to be

$\frac{u}{U_\infty} = A + By + Cy^2$

With three conditions

$u = 0\ at\ y = 0; u = U_\infty\ at\ y = \delta;\ \frac{\partial u}{\partial y} = 0\ at\ y = \delta\ \therefore A = 0$

$1 = A + B\delta + C\delta^2 = B\delta + C\delta^2$;

$0 = B + C * 2\delta$

Then A=0; $B = \frac{2}{\delta}$; $C = -\frac{1}{\delta^2}$. The velocity profile is

$$\frac{u}{U_\infty} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2} - - - - - - - (a)$$

This is substituted into Von Karman's integral equation (6) to obtain

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right] dy$$

$$= \frac{d}{dx}\int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \frac{d}{dx}\left[\frac{2y^2}{2\delta} - \frac{4y^3}{3\delta^2} + \frac{2y^4}{4\delta^3} - \frac{y^3}{3\delta^2} + \frac{2y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$= \frac{d}{dx}\left[\delta - \frac{4}{3}\delta + \frac{1}{2}\delta - \frac{1}{3}\delta + \frac{1}{2}\delta - \frac{1}{5}\delta\right]$$

$$= \frac{d}{dx}\left[1 - \frac{4}{3} + \frac{1}{2} - \frac{1}{3} + \frac{1}{2} - \frac{1}{5}\right]\delta$$

$$= \frac{d}{dx}\left(\frac{30-25-3}{15}\right)\delta = \frac{d}{dx}\frac{2}{15}\delta$$

$$\therefore \ \tau_0 = \frac{2}{15}\rho U_\infty^2 \frac{d\delta}{dx} \qquad - - - - - (b)$$

We also use $\tau_0 = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$

From Eq. (a)

$$\tau_0 = \mu\, U_\infty \frac{2}{\delta} \qquad - - - - - (c)$$

Equating Eq's (b & c) we obtain

$$\frac{2}{15}\rho U_\infty^2 \frac{d\delta}{dx} = \mu U_\infty \frac{2}{\delta}$$

$\delta\, d\delta = 15\frac{\nu}{U_\infty}\, dx$ Using $\delta = 0 \ at\ x = 0 \ \ after\ integration$

$$\int_0^\delta \delta d\delta = \int_0^x 15\frac{\nu}{U_\infty}\, dx$$

$$\frac{\delta^2}{2} = 15\frac{\nu}{U_\infty}\, x$$

$$\therefore \ \delta = 5.48\sqrt{\frac{\nu x}{U_\infty}} \qquad - - - - (d)$$

This is 18% higher than the value using the cubic profile, the wall shear is found to be

$\tau_0 = \frac{2\mu U_\infty}{\delta}$ Substitute Eq. (d)

$$\tau_0 = \frac{2\mu U_\infty}{5.43}\sqrt{\frac{U_\infty}{\nu x}}$$

$$\tau_0 = 0.365\,\rho U_\infty^2\sqrt{\frac{\nu}{x\,U_\infty}} = \frac{0.365\rho U_\infty^2}{\sqrt{Re_x}}$$

This is 13% higher than the value using the cubic velocity profile.

3- Solution of Turbulent B.L. Power – Law Form.

The power– law form is

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/n} \qquad n = \begin{cases} 7 & Re_x < 10^7 \\ 8 & 10^7 < Re_x \le 10^8 \\ 9 & 10^8 < Re_x \le 10^9 \end{cases} \qquad (21)$$

The Balsius formula is an empirical relation for the local friction coefficient

$$C_f = 0.046 \left(\frac{\nu}{U_\infty \delta}\right)^{\frac{1}{4}} \qquad (22)$$

We have $C_f = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2}$ $\quad --\rightarrow \quad \tau_0 = C_f \frac{1}{2} \rho U_\infty^2$

$$\therefore \; \tau_0 = 0.023\, \rho U_\infty^2 \left(\frac{\nu}{U_\infty \delta}\right)^{1/4} \qquad (23)$$

Substitute the velocity profile in Von Karman's integral Eq. 6 with $Re_x < 16^7$

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx} \int_0^\delta \left[\left(\frac{y}{\delta}\right)^{1/7} \left(1 - \left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right)\right] dy$$

$$\tau_0 = \frac{7}{72}\, \rho U_\infty^2 \frac{d\delta}{dx} \qquad (24)$$

Equating Eq's (23 & 24) for τ_0, we find that

$$\delta^{\frac{1}{4}}\, d\delta = 0.237 \left(\frac{\nu}{U_\infty}\right)^{\frac{1}{4}} dx$$

Assuming a turbulent flow from the leading edge L>>x_L , from integration the above eqn.

$$\frac{\delta^{\frac{5}{4}}}{\frac{5}{4}} = 0.237 \left(\frac{\nu}{U_\infty}\right)^{1/4} x$$

After taking the root (4/5) and multiplied by $(x/x)^{1/5}$ gives the following

$$\delta = 0.38\, x\, (Re_x)^{-1/5} \qquad (25)$$

Substituting Eq. 8.30 for δ in to Eq. (22) we find that

$$C_f = 0.059 (R_e)^{-1/5} \quad for \quad Re \; < 10^7 \qquad (26)$$

The drag force = $F_D = \tau_0\, A = C_f \frac{1}{2}\, \rho U_\infty^2 (LW)$ (27)

Where W is the width of plate and C_f from Eq. (26).

Ex.2

Estimate the boundary layer thickness at the end of a 4-m-long flat surface if the free-stream velocity $U_\infty = 5 \frac{m}{s}$. Use atmospheric air at $30C^\circ$. And predict the drag force if the surface is 5m wide

a) Neglect the laminar portion of the flow
b) Account for the Laminar portion using $Re_{crit.} = 5 * 10^5$.

Sol.

a) Let us first assume turbulent flow from the leading edge. The B.L thickness is given by Eq. (25). It is

$$\delta = 0.38\, x\, {Re_x}^{-1/5}$$

$$\delta = 0.38 * 4 * \left(\frac{5*4}{1.6*10^{-5}}\right)^{-1/5} = 0.092m$$

The drag force is using Eq's (8.31 & 8.32); $Re = \frac{U_\infty . L}{\nu}$

$$F_D = C_f * \frac{1}{2} \rho U_\infty^2 (LW) = 0.059 (Re)^{-1/5} * \frac{1}{2} \rho U_\infty^2 (L.W)$$

$$F_D = 0.059 \left(\frac{5*4}{1.6*10^{-5}}\right)^{-1/5} * \frac{1}{2} * 1.16 * 5^2 * 4 * 5 = 1.032N$$

$Re_L = \frac{5*4}{1.6*10^{-5}} = 1.25 * 10^6$. Hence, the calculation is acceptable

The distance is found as follows

$Re_{crit.} = 5 * 10^5 = \frac{U_\infty x_L}{\nu}$

$\therefore\ x_L = \frac{5*10^5*1.6*10^{-5}}{5} = 1.6\ m$

The B.L thickness at x_L is found from Eq. (16) with

$\delta = 4.65 \frac{x_L}{\sqrt{Re_x}} = \frac{4.65*1.6}{\sqrt{5*10^5}} = 0.0105\ m$

To find the origin of turbulent flow, using equation of B.L thickness in turbulent as

$\delta = 0.38\ x\ (Re_x)^{-1/5} = \frac{0.38x\prime}{\left(\frac{U_\infty x}{\nu}\right)^{\frac{1}{5}}} = \frac{0.38x^{\frac{4}{5}}}{\left(\frac{U_\infty}{\nu}\right)^{\frac{1}{5}}}$

$\therefore\ x'^{4/5} = \frac{\delta}{0.38}\ (\frac{U_\infty}{V})^{1/5}$ where $\delta =$ the same at end of L.B

$x' = \left(\frac{0.0105}{0.38}\right)^{5/4}\ \left(\frac{5}{1.6*10^{-5}}\right)^{1/4} = 0.2663m$

The distance x_t as in figure is then

$x_t = L - x_L + x' = 4 - 1.6 + 0.266 = 2.666M$

To find the B.L thickness at the end of plate using Eq. 8.30

$\delta = 0.38\ x\ Re^{-1/5} = 0.38 * 2.666 * \left(\frac{5*2.666}{1.6*10^{-5}}\right)^{-1/5}$

$\delta = 0.0662m$

The value of part (a) is 28% to high when compared with this more accurate value.