Lecture-Twelve

Centrifugal Pump Theory

1- Pump Head.

The main assumption in this analysis the flow is to be steady, the pump basically increases the Bernoulli head of the flow between the eye, point (1), and the exit, point (2) as show in Fig. (1)

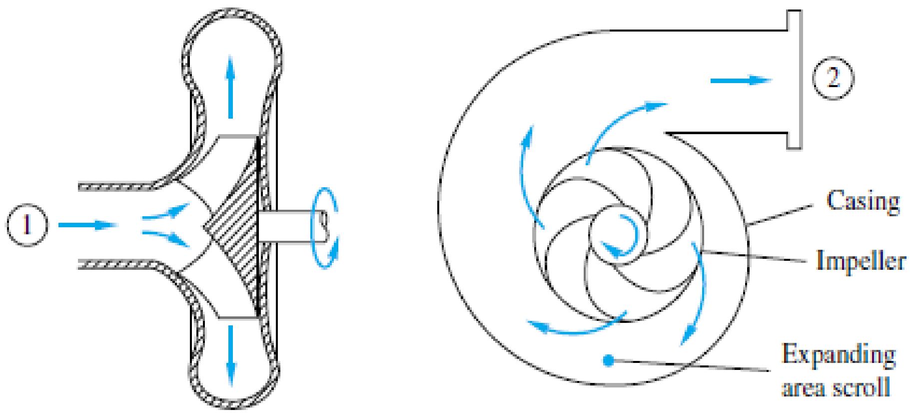


Figure 1: Schematic of a typical centrifugal pump.

The change in head is denoted by (*H*).

$$H = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + Z\right)_2 - \left(\frac{p}{\rho g} + \frac{V^2}{2g} + Z\right)_1 = h_s - h_f \qquad (1)$$

Where h_s is the pump head supplied and h_f is the losses. Now, the primary output parameter for any turbomachine is the net head (*H*).

Usually $V_2 \cong V_1$ and $(Z_2 - Z_1)$ less than one meter, then, the net pump head is

$$H = \frac{p_2 - p_1}{\rho g} = \frac{\Delta p}{\rho g} \qquad (2)$$

The power delivered to the fluid simply equals the specific weight times the discharge times the net head change.

$$P_w = \rho g Q H \qquad (3)$$

This is called the water horsepower. The power required to drive the pump is the brake horsepower

$$P_{bhp} = \omega T_{sh} \qquad (4)$$

Where ω is the shaft angular velocity.

T_{sh} is the shaft torque.

2- *Pump Theory.*

To predict actually the head, power, efficiency and flow rate of a pump two theoretical approaches are possible

- Pump Theory
  - Simple one-dimensional formula.
  - Complex digital – computer models.

- To construct an elementary theory of pump performance in this chapter, we assume the following
  - One-dimensional flow.
  - Idealized fluid – velocity vector through the impeller.
  - Angular momentum theorem for C.V.

To analyze the centrifugal pump, we choose the annular region that encloses the impeller section as the control volume C.V. as in Fig. (2), which shows the idealized velocity diagram and the impeller geometry.

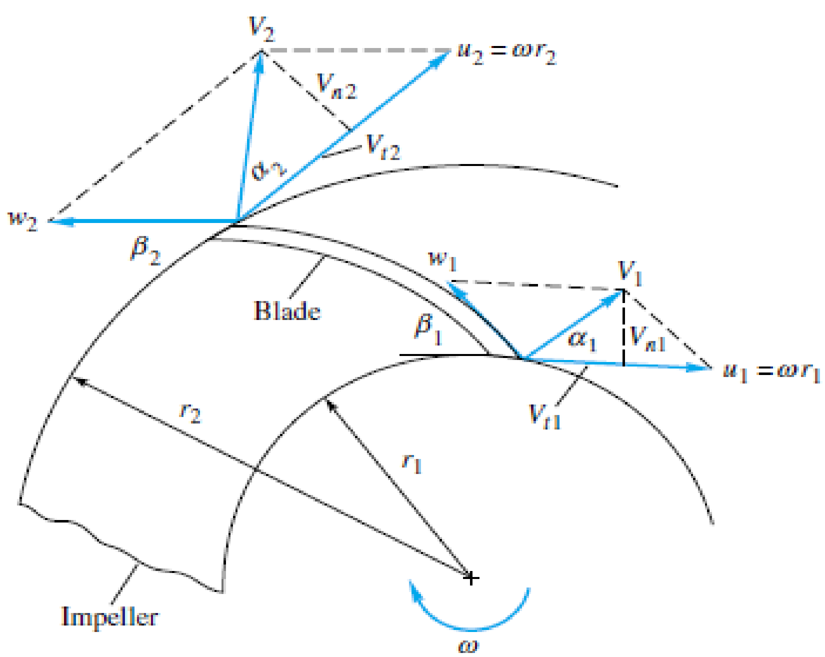


Figure 2: Inlet and exit velocity diagrams for an idealized pump impeller.

Since, r_1 inside radius of impeller.
r_2 outside radius of impeller.
ω The angular velocity of shaft impeller blades.

At Inlet,
$u_1 = \omega r_1$ is the circumferential speed of the tip impeller at r_1.
w_1 velocity component of fluid tangent or parallel to the blade angle β_1.
V_1 is the absolute velocity of fluid at entrance, is the vector sum of w_1 and u_1.

At Exit,
$u_2 = \omega r_2$ is the circumferential speed of the tip impeller at r_2.
w_2 velocity component outlet of fluid tangent or parallel to the blade angle β_2.
V_2 is the absolute velocity of fluid at exit, is the vector sum of w_2 and u_2.

The angular – momentum theorem to a radial flow devices (sec. 4.7, 1st semester), we arrived to a result for the applied torque T_{sh}

$$T_{sh} = \dot{m}\,(r_2V_{t2} - r_1V_{t1}) = \rho Q(r_2V_{t2} - r_1V_{t1}) \qquad (5)$$

Where V_{t1} and V_{t2} are the absolute circumferential or tangent velocity components of the flow at inlet and exit. The power delivered to the fluid is thus,

$$P_w = \omega T_{sh} = \rho Q(u_2V_{t2} - u_1V_{t1}) \qquad (6)$$

Or $H = \frac{P_w}{\rho g Q} = \frac{1}{g}(u_2V_{t2} - u_1V_{t1})$ (7)

Eq's (5, 6 and 7) are the Euler turbomachine equations, showing that the torque, power and ideal head are functions only of the rotor-tip velocities u_1, u_2 and the absolute fluid tangential velocities V_{t1} and V_{t2} independent of the axial velocities.

We can rewriting these relation in other form, from the geometry of Fig. (6)

$$V^2 = u^2 + w^2 - 2uw\cos\beta \quad ; \quad w\cos\beta = u - V_t$$

Or $uV_t = \frac{1}{2}(V^2 + u^2 - w^2)$ (8)

Substituting Eq. (8) into Eq. (7) gives

$$H = \frac{1}{2g}\left[(V_2^2 - V_1^2) + (u_2^2 - u_1^2) - (w_2^2 - w_1^2)\right] \qquad (9)$$

Substituting for *(H)* from its definition in Eq. (1) and rearranging, we obtain the classic relation

$$\frac{p}{\rho g} + Z + \frac{w^2}{2g} - \frac{r^2\omega^2}{2g} = constant \qquad (10)$$

This is the Bernoulli equation in rotating coordinates and applies to either two or three-dimensional ideal incompressible flow. For a centrifugal pump, the power can be related to the radial velocity $V_n = V_t \tan\alpha$ and the continuity equation

$$P_w = \rho Q(u_2V_{n2}\cot\alpha_2 - u_1V_{n1}\cot\alpha_1) \qquad (11)$$

Where

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} \quad \text{and} \quad V_{n1} = \frac{Q}{2\pi r_1 b_1} \qquad (12)$$

Where b_1 and b_2 are the blade widths at inlet and exit, with the pump parameters (r_1, r_2, β_1, β_2 and ω) are known. Eq. (7) or Eq. (11) is used to compute idealized power and head versus discharge.

The 'design' flow rate Q^* is commonly estimated by using that the flow enters exactly normal to the impeller; $\alpha_1 = 90^\circ$; $V_{n1} = V_1$

Ex.1

Given are the following data for a commercial centrifugal water pump: r_1=4 in, r_2= 7 in, β_1=30°, β_2 =20°, speed N=1440 r/min. Estimate

a- The design – point discharge.
b- The water horsepower.
c- The head if b_1=b_2=1.75 in.

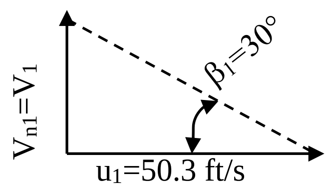
Sol.

(a) $\omega = \frac{2\pi N}{60} = \frac{2\pi(1440)}{60} = 150.8\, rad/s$

$u_1=\omega r_1$=150.8 (4/12) = 50.3 ft/s

$u_2=\omega r_2$=150.8 (7/12) = 88.0 ft/s

For design point $V_{n1}=V_1$ and $\alpha_1=90^\circ$ then the inlet velocity diagram as follows



$$\tan 30 = \frac{V_{n1}}{u_1} \quad \rightarrow \quad V_{n1} = u_1 * tan30 = 29.0\, ft/s$$

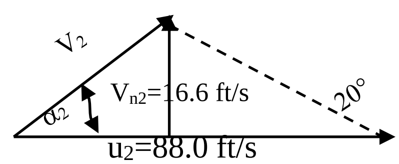
Hence the discharge is

$$Q = 2\pi r_1 b_1 V_{n1} = 2\pi\left(\frac{4}{12}\right)\left(\frac{1.75}{12}\right)(29.0) = 8.87\,\frac{ft^3}{s}$$

(b) The outlet radial velocity follows from Q

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = \frac{8.87}{2\pi(\frac{7}{12})(\frac{1.75}{12})} = 16.6\, ft/s$$

This enable us to construct the outlet – velocity diagram as in below



The tangential component is

$$V_{t2} = u_2 - V_{n2}\cot\beta_2 = 88.0 - 16.6\, cot20° = 42.4\, ft/s$$

$$\propto_2 = tan^{-1}\frac{16.6}{42.4} = 21.4°$$

The power is then computed as follows, with V_{t1} = 0 at the design point.

$$P_w = \rho Q u_2 V_{t2} = (1.94 * 8.87 * 88.0 * 42.4) = 64100\, ft\, lb/s$$

$$P_w = \frac{64100}{550} = 117\, hp$$

(c) Finally, the head is estimated by the following

$$H \approx \frac{P_w}{\rho g Q} = \frac{64100}{(62.4)(8.87)} = 116\, ft$$

3- Pressure Developed by the Impeller.

Fig. (3) shows the general system arrangement of a centrifugal pump.

Z_s Suction level above the water level.

Z_d Delivery level above the impeller outlet.

h_{fs}, h_{fd} Friction losses head in both suction and delivery sides.

V_s, V_d Fluid velocities in pipes for both sides.

Applying B.E. between the water level and pump suction.

$$\frac{p_s}{\gamma} + Z_s + \frac{V_s^2}{2g} + h_{fs} = \frac{p_a}{\gamma} \qquad (13)$$

$$\therefore \frac{p_s}{\gamma} = \frac{p_a}{\gamma} - Z_s - \frac{V_s^2}{2g} - h_{fs} \qquad (14)$$

Similarly applying B.E. theorem between the pump delivery and the delivery at the tank,

$$\frac{p_d}{\gamma} + \frac{V_d^2}{2g} = \frac{p_a}{\gamma} + Z_d + \frac{V_d^2}{2g} + h_{fd}$$

$$or\ \frac{p_d}{\gamma} = \frac{p_a}{\gamma} + Z_d + h_{fd} \qquad (15)$$

Where p_d is the pressure at the pump delivery, from Eq.(14 &15)

$$\frac{p_d}{\gamma} - \frac{p_s}{\gamma} = \left(\frac{p_a}{\gamma} + Z_d + h_{fd}\right) - \left(\frac{p_a}{\gamma} - Z_s - \frac{V_s^2}{2g} - h_{fs}\right)$$

$$\frac{p_d - p_s}{\gamma} = Z_d + Z_s + \frac{V_s^2}{2g} + h_L = H_e + \frac{V_s^2}{2g} \qquad (16)$$

Where H_e is the effective head and h_L is the total friction head.

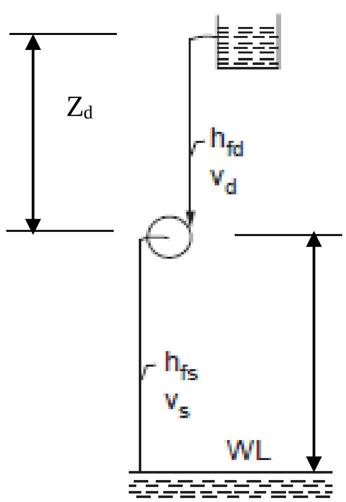


Figure 3: Centrifugal pump system

4- Manometric Head.

The official code defines the head on the pump as the difference in total energy head at the suction and delivery flanges. This head is defined as manometric head. The total energy at suction inlet or suction side expressed as suction head of fluid and termed as H_s

$$H_s = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + Z_{sd}$$

Where Z_{sd} is the height of suction gauge from datum. The total energy at the delivery side of the pump expressed as delivery head and termed as H_d

$$H_d = \frac{p_d}{\gamma} + \frac{V_d^2}{2g} + Z_{dd}$$

Where Z_{dd} is the height of delivery gauge from datum. Know, the difference in total energy of fluid is defined as H_m

$$H_m = H_d - H_s = \left(\frac{p_d}{\gamma} - \frac{p_s}{\gamma}\right) + \left(\frac{V_d^2 - V_s^2}{2g}\right) + (Z_{dd} - Z_{sd}) \qquad (17)$$

Substituting Eq.(16) in Eq. (17) and rearranging will be as follows

$$H_m = H_e + \frac{V_d^2}{2g} + (Z_{dd} - Z_{sd}) \qquad (18)$$

As $(Z_{dd} - Z_{sd})$ is small and $\frac{V_d^2}{2g}$ is also small as the gauges are fixed as close as possible.

$$\therefore\ H_m = static\ head + all\ losses \qquad (19)$$