Lecture-Thirteen

Centrifugal Pump Performance

1- *Pump Efficiency*.

When the losses are exists in the pump, then (***P***P_w) is actually less than (P_{bhp}). The goal of the pump designer is to make the efficiency ***(η)*** as high as possible over as broad a range of discharge (Q) as possible. The efficiency can be divided to three types as follows

- Pump Efficiency
  - Manometric Efficiency
  - Mechanical Efficiency
  - Volumetric Efficiency

A- *Manometric efficiency.*

The ideal head ***(H)*** can be expressed by Eq. (7) [L-2]

$$H_{ideal} = \frac{1}{g}(u_2 V_{t2} - u_1 V_{t1})$$

From Fig. (1), the inlet tangent velocity is generally equal to zero due to no guide vanes at inlet. So the inlet triangle is right angle as shown in Fig.(1.a). Therefor

$V_1 = V_{n1}$ and are vertical

$$tan\beta_1 = \frac{V_1}{u_1}, \quad or\ , = \frac{V_{n1}}{u_1}$$

Since, $V_{t1} = 0.0$

$$H_{ideal} = \frac{u_2 V_{t2}}{g} \qquad (1)$$

From the outlet triangle

$$u_2 = \omega r_2 = \frac{2\pi N}{60} r_2 = \frac{\pi d_2 N}{60}$$

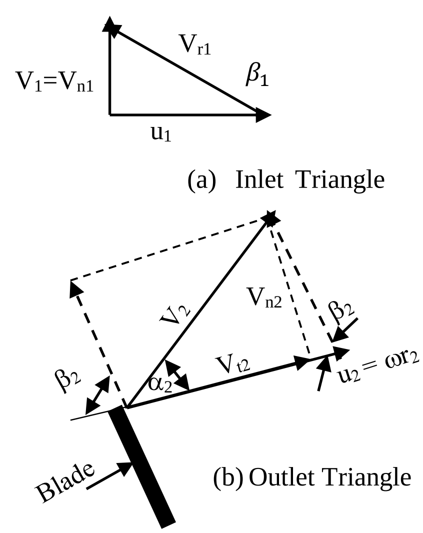
$$tan\beta_2 = \frac{V_{n2}}{u_2 - V_{t2}}$$

$$u_2 - V_{t2} = \frac{V_{n2}}{tan\beta_2}$$

$$\therefore V_{t2} = u_2 - \frac{V_{n2}}{tan\beta_2}$$

From Eq. (9.20)

$$\therefore H_{ideal} = \frac{u_2}{g}\left(u_2 - \frac{V_{n2}}{tan\beta_2}\right)$$



(a) Inlet Triangle

(b) Outlet Triangle

Figure 1: Velocity triangles for backward curved bladed pump.

Now, manometric efficiency is defined as the ratio of manometic head and ideal head

$$\eta_m = \frac{H_m}{H_{ideal}} = \frac{H_m\, g}{u_2 V_{t2}} = \frac{H_m\, g}{u_2\left(u_2 - \frac{V_{n2}}{tan\beta_2}\right)} \qquad (2)$$

B- *Mechanical Efficiency.*

The mechanical efficiency is defined as

$$\eta_{mech} = \frac{Energy\ transfer\ to\ the\ fluid}{Work\ input}$$

$$\eta_{mech} = \frac{\rho Q(u_2 V_{t2})}{Power\ input} = \frac{\rho Q(u_2 V_{t2})}{P_{bhp}} \qquad (3)$$

C- *Volumetric Efficiency.*

There are always some leakage after being imparted energy by the impeller.

$$\text{Volumetric efficiency} = \frac{\text{Volum deliverd (Q)}}{\text{Volume passing throught impeller (Q+Q}_L)}$$

Where (Q_L) is the losses of fluid due to leakage, thus, the total efficiency is simply the product of its three parts.

$\eta_o = \eta_m \eta_{mech} \eta_{vol.}$ Or from basic definition can be defined as

$$\eta_o = \frac{P_w}{P_{bhp}} = \frac{\rho g Q H}{P_{bhp}} \qquad (4)$$

Ex.1

The following details refer to a centrifugal pump. Outer diameter 30cm, eye diameter 15 cm. blade angle at inlet (β_1=30°) blade angle at outlet (β_2=25°). Impeller speed (N=1450 rpm). The velocity remains constant. The whirl (tangent) velocity at inlet is zero. Determine (a)- the torque applied if the (η_m=0.76%). (b)- The power when the width of blades at outlet is 2 cm and equal the width at inlet. (c)- The head.

Sol.

$$u_2 = \frac{2\pi r_2 N}{60} = \frac{\pi d_2 N}{60} = \frac{\pi * 0.3 * 1450}{60} = 22.78\ m/s$$

$$u_1 = \frac{2\pi r_1 N}{60} = \frac{\pi d_1 N}{60} = \frac{\pi * 0.15 * 1450}{60} = 11.39\ m/s$$

$V_1 = V_{n1}$ since $V_{t1} = 0$

From inlet velocity diagram

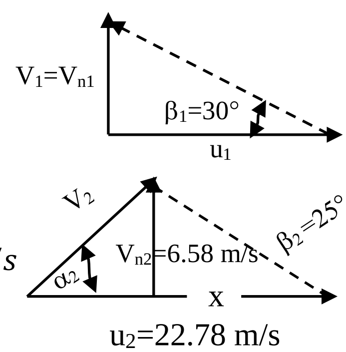
$$\tan 30 = \frac{V_{n1}}{u_1} \rightarrow \ V_{n1} = u_1 * tan30 = 11.39 * tan30 = 6.58\ m/s$$

$V_{t2} = u_2 - x$

$$tan\beta_2 = \frac{V_{n2}}{X}$$

$$\therefore X = \frac{V_{n2}}{tan\beta_2}$$

$$\therefore V_{t2} = u_2 - \frac{V_{n2}}{tan\beta_2} = 22.78 - \frac{6.58}{\tan 25} = 8.67\ m/s$$



Power delivered to the fluid as

$$P_w = \omega T_{sh} = \rho Q(u_2 V_{t2} - u_1 V_{t1})$$

$$Q = 2\pi * r_2 * b * V_{n2} = \pi * d_2 * b * V_{n2} = \pi * 0.3 * 0.02 * 6.58 = 0.124\ m^3/s$$

(a).

$$T_{sh} = \dot{m}\,(r_2 V_{t2} - r_1 V_{t1}) = \ \rho Q(r_2 V_{t2} - r_1 V_{t1})$$

$$T_{sh} = \rho * \pi * d_2 * b * V_{n2}(r_2 V_{t2} - r_1 V_{t1}) = 1000 * \pi * 0.3 * 0.02 * 6.58 * (0.15 * 8.67 - 0.0) = 161\ N.m$$

(b)

$$P_w = 1000 * 0.124 * (22.78 * 8.67 - 11.39 * 0.0) = 24490\ W$$

$$P_w = \frac{P_w}{746} = \frac{24490}{746} \approx 33\ hp$$

$$\eta_m = \frac{P_w}{P_{bhp}} \quad \rightarrow P_{bhp} = \frac{P_w}{\eta_m} = \frac{33}{0.76} \approx 44\ hp$$

(c)

Head (*H*) as

$$H = \frac{P_w}{\rho g Q} = \frac{1}{g}(u_2 V_{t2} - u_1 V_{t1}) = \frac{1}{9.81}(22.78 * 8.67 - 0.0) = 20\ m$$

2- *Pump Performance Curves.*

Performance charts are always plotted for constant shaft-rotation speed ***N*** (rpm). The basic independent variable is taken to be discharge ***Q*** in (gal /min or m^3/s). The dependent variables or output, are taken to be head ***H*** for liquid (pressure rise ***Δp*** for gases), brake horsepower P_{php} and efficiency $\boldsymbol{\eta}$. Fig.(2) shows typical performance curves for centrifugal pump.

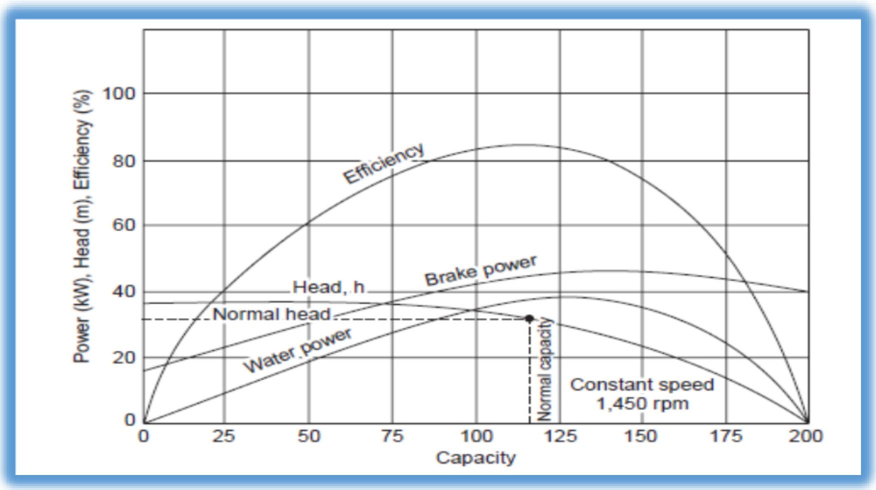


Figure 2: Centrifugal pump characteristics at constant speed.

The efficiency $\boldsymbol{\eta}$ is always zero at no flow and $\boldsymbol{Q_{max}}$, and it reaches a maximum perhaps 80 to 90 percent at about $\boldsymbol{0.6Q_{max}}$. This is the design point flow rate Q^* or best efficiency point (BEP) and $\boldsymbol{\eta}_{max.}$. The head and horsepower at BEP will be termed H^* and P^*_{bhp}.

3- *Net Positive – Suction Head (NPSH).*

NPSH, which is the head required at the pump inlet to keep the liquid from cavitation or boiling. The *NPSH* is defined as

$$NPSH = \frac{p_s}{\rho g} + \frac{V_s^2}{2g} - \frac{p_v}{\rho g} \qquad (5)$$

Where (p_s &V_s) are the pressure and velocity at the pump inlet and p_v is the vapor pressure of the liquid. The right–hand side is equal or greater than (*NPSH*) in the actual system to avoid cavitation. Applying the energy equation between sump surface and the pump suction level.

$$\frac{p_s}{\gamma}+\frac{V_s^2}{2g}+Z_s+h_{fs}=\frac{p_a}{\gamma} \qquad (6)$$

From Eq's (5&6)

$$NPSH=\frac{p_a}{\gamma}-\frac{p_v}{\gamma}-Z_s-h_{fs} \qquad (7)$$

Thoma cavitation parameter is defined by

$$\sigma=\frac{(NPSH)}{H}=\frac{\left(p_a/\gamma\right)-\left(p_v/\gamma\right)-Z_s-h_{fs}}{H}$$

At cavitation conditions

$$\sigma=\sigma_c \ and\ \frac{p_s}{\gamma}=\frac{p_v}{\gamma}$$

$$\sigma_c=\frac{\left(p_a/\gamma\right)-\left(p_v/\gamma\right)-Z_s-h_{fs}}{H}=\frac{V_s^2}{2gH} \qquad (8)$$

The height of suction, the frictional losses in the suction line play an important role for avoiding cavitation at a location.

4- *Outlet Blade Angles and Specific Speed.*

Different blade arrangements can be used in design of centrifugal pumps. Three possible orientations of outlet blade angle. Forward curved blade (β_2 >90º), radial curved blade (β_2=90º) and backward curved blade (β_2 < 90º). Fig. (3) are shown the velocity triangles for three arrangements.

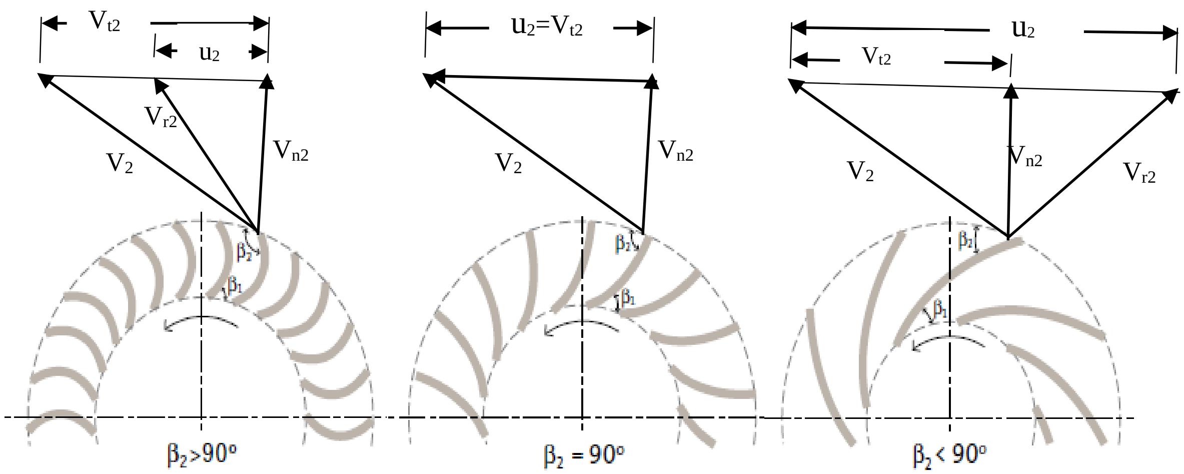


Figure 3: Blade shape arrangements with outlet velocity triangles

The dimensionless parameter (N_s) is known as the specific speed of pumps. In practice is used as

$$N_s=\frac{N\sqrt{Q}}{H^{3/4}} \qquad (9)$$

The (N_s) quantity have been derived from dimensional analysis technique, and defined as the speed at which the pump will operate to deliver unit flow under unit head.

Ex.2

The outer diameter and width of a centrifugal pump impeller are 55 cm and 3 cm. The pump runs at 1300 rpm. The suction head is 7 m and the delivery head is 45 m. The frictional drops in suction side is 2.5 m and in the delivery side is 9 m. The blade angle at out let is 33°. The manometric efficiency is 83% and the overall efficiency is 77%. Determine

a- The power required to drive the pump.

b- Calculate the pressures at the suction and delivery side of the pump.

Sol.

Assume the inlet tangential velocity is zero, the total head against the pump is

H = 45+7+2.5+9= 63.5 m

$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi * 0.55 * 1300}{60} = 37.4\,\frac{m}{s}$; $since \;\; u_2 = \omega r_2$

$\eta_m = \frac{gH}{u_2 V_{t2}} = 0.83 = \frac{9.81*63.5}{37.4*V_{t2}}$ $solving\ for\ V_{t2} = 20.0\ m/s$

The outlet velocity triangle is used to calculate V_{n2},

$tan\beta_2 = \frac{V_{n2}}{u_2 - V_{t2}}$

$\therefore\ V_{n2} = tan\beta_2 (u_2 - V_{t2}) = tan33(37.4 - 20.0) = 11.36\ m/s$

Flow rate Q is

$$Q = \pi * d_2 * b_2 * V_{n2} = \pi * 0.55 * 0.03 * 11.36 = 0.58886\ m^3/s$$

$$\eta_o = \frac{P_w}{P_{bhp}} \quad Solving\ for\ \ Power\ (P_{bhp}) = \frac{P_w}{\eta_o} = \frac{\rho g Q H}{\eta_o}$$

$$\therefore\ P_{bhp} = \frac{1000 * 9.81 * 0.58886 * 63.5}{0.77} = 476391.56\ W$$

Now, consider the water level and the suction level are (1 & 2)

$\frac{p_a}{\gamma} + \frac{V_1{}^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2{}^2}{2g} + Z_2 + losses$

$\frac{101000}{9810} + 0 + 0 = \frac{p_2}{9810} + \frac{11.36^2}{2 * 9.81} + 7 + 2.5 \quad solving\ for\ {}^{p_2}/_{\gamma}$

${}^{p_2}/_{\gamma} = 1.218\ m\ absolute$

Consider suction side and delivery side as 2 and 3,

$\frac{p_2}{\gamma} + \frac{V_2{}^2}{2g} + H_{ideal} = \frac{p_3}{\gamma} + \frac{V_3{}^2}{2g} + Z_3$ $;since\ H_{ideal} = \frac{u_2 V_{t2}}{g}$

$V_3 = \sqrt{V_{t2}^2 + V_{n2}^2} = \sqrt{20^2 + 11.36^2} = 23\ m/s$

$$1.218 + \frac{11.36^2}{2 * 9.81} + \frac{37.4 * 20}{9.81} = \frac{p_3}{\gamma} + \frac{23^2}{2 * 9.81};solving\ for$$

$$\frac{p_3}{\gamma} = 57\ m\ \ absolute.$$

Ex.3

A centrifugal pump was tested for cavitation initiation. Total head was 40 m and flow rate was 0.06 m^3/s. Cavitation started when the total head at the suction side was 3 m. The atmospheric pressure was 760 mm Hg and the vapor pressure at this temperature was 2 kPa. It was proposed to install the pump where the atmospheric pressure is 700 mm Hg and the vapor pressure at the location temperature is 1 kPa. If the pump develops the same total head and flow, can the pump be fixed as the same height as the lab setup? **What should be the new height?**

Sol.

At the suction point

Total head = Vapour pressure + velocity head.

∴ Velocity head = Total head – Vapor pressure in head of water

$\frac{V_s^2}{2g} = 3 - \frac{2*10^3}{10^3*9.81} = 2.796\ m$

Cavitation parameter (σ) is defined by

$\sigma = \frac{V_s^2}{2*g*H} = \frac{2.796}{40} = 0.0699$

Applying B.E. between water level and suction point,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{p_{atm}}{\gamma}-h_f$$

$$\left(Z_1+h_f\right)=\frac{p_{atm}}{\gamma}-\sigma H-\frac{p_v}{\gamma}=\left(\frac{760*13.6}{1000}\right)-2.796-\frac{2*10^3}{10^3*9.81}=7.336\ m$$

At the new location (head and flow rate being the same, friction loss will be the same)

$$\left(Z_1{}'+h_f{}'\right)=\frac{p_{atm}}{\gamma}-\sigma H-\frac{p_v}{\gamma}=\left(\frac{700*13.6}{1000}\right)-2.796-\frac{1*10^3}{10^3*9.81}=6.622\ m$$

$$h_f=h_f'$$

$$\therefore\ (Z_1-Z_1')=0.714\ m$$

The pump should be lowered by 0.714 m, since the new height is 6.622 m.