and,

$$\begin{aligned}\dot{y} &= -\,y\left(\sin x \ln(1-3x) + \cos x \frac{3}{1-3x}\right) \\ &= -\,(1-3x)^{\cos x}\left(\sin x \ln(1-3x) + \cos x \frac{3}{1-3x}\right)\end{aligned}$$

3.11 Exercises

1. An object is dropped from the top of a 100 m high tower. Its height above ground after t sec is $100 - 4.9t^2$ m. How fast is it falling 2 sec after it is dropped?

2. At t sec after liftoff, the height of a rocket is $3t^2$ ft. How fast is the rocket climbing 10 sec after liftoff?

3. Using the definition of the derivative,
   1- Differentiate the following functions,
   2- Find the tangent equations,
   3- Find the values of the derivatives as specified,

   (a) $f(x) = x^3 + 2x$, $a = 0$, $\dot{f}(-2)$
   (b) $f(x) = x + \frac{5}{x}$, $a = 1$, $\dot{f}(0)$
   (c) $f(x) = x + \sin(2x)$, $a = \frac{\pi}{2}$, $\dot{f}(\pi)$
   (d) $f(x) = \cos x + 4\sin(2x)$, $a = \pi$, $\dot{f}(\frac{\pi}{2})$

4. Does the parabola $y = 2x^2 - 13x + 5$ have a tangent whose slope is -1? If so, find an equation for the line and the point of tangency. If not, why not?

5. Find the derivative of the following functions,
   (a) $y = \frac{1}{x}(x^2 + \mathbf{e}^x)$
   (b) $y = (x^2 + 1)(x^3 + 3)$
   (c) $y = \frac{t^2-1}{t^3+1}$
   (d) $y = \mathbf{e}^{-x}$
   (e) $y = \sqrt[7]{x^2} - x^2$

6. Find the second derivative of the following functions,
   (a) $y = \frac{4x^3}{3} - x + 2\mathbf{e}^x$
   (b) $y = \frac{x^3}{3} + \frac{x^2}{2} + \mathbf{e}^x$
   (c) $y = x^3\mathbf{e}^x$
   (d) $y = \frac{(x+1)(x+2)}{(x-1)(x-2)}$

7. If gas in a cylinder is maintained at a constant temperature T, the pressure P is related to the volume V by a formula of the form $P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$ in which a, b, n and R are constants. Find $\frac{dP}{dV}$.

8. A dynamite blast blows a heavy rock straight up with a launch velocity of 160 m/sec. It reaches a height of $s = 160t - 16t^2$ m after t sec.
   (a) How high does the rock go?
   (b) What are the velocity and speed of the rock when it is 256 m above the ground on the way up? On the way down?
   (c) What is the acceleration of the rock at any time t during its flight (after the blast)?
   (d) When does the rock hit the ground again?.

9. Suppose that the distance an aircraft travels along a runway before takeoff is given by $D = \frac{10}{9}t^2$, where D is measured in meters from the starting point and t is measured in seconds from the time the brakes are released. The aircraft will become airborne when its speed reaches 200 km/h. How long will it take to become airborne, and what distance will it travel in that time?

10. Find the derivative of the following functions,
    (a) $y = \frac{\cos x}{\sin^2 x}$
    (b) $y = x^2 \cot x - \frac{1}{x^2}$
    (c) $y = x\mathbf{e}^{-x} \sec x$
    (d) $y = \frac{4}{\cos x} + \frac{1}{\tan x}$

11. Find the derivative of the following functions,
    (a) $y = (2x+1)^5$
    (b) $y = \sec(\tan x)$
    (c) $y = \mathbf{e}^{\cos^2(\pi t - 1)}$

(d) $y = \tan^2(\sin^3 t)$

12. Find the derivative of the following functions,
    (a) $\tan^{-1} 1$
    (b) $\sec^{-1}(-2)$
    (c) $\sin(\cos^{-1} \frac{\sqrt{2}}{2})$
    (d) $y = \cot^{-1} \frac{1}{x} - \tan^{-1} x$
    (e) $y = \ln(x^2 + 4) - x \tan^{-1} \frac{x}{2}$

13. Find the derivative of the following functions (use implicit differentiation),
    (a) $x^2y + xy^2 = 6$
    (b) $xy = \cot(xy)$
    (c) $x \cos(2x + 3y) = y \sin x$
    (d) $\mathbf{e}^{2x} = \sin(x + 3y)$

14. Find the derivative of the following functions,
    (a) $y = \ln 3x + x$
    (b) $y = \ln(x + 1) - \mathbf{e}^x$
    (c) $y = \sqrt{x(x + 1)}$
    (d) $y = \sin x\sqrt{x + 3}$
    (e) $y = \sqrt[3]{\frac{x(x-2)}{x^2+1}}$
    (f) $y = 3 \log_8(\log_2 t)$