Lecture Two

Heat Exchanger Analysis

1- Introduction.

For designing or predicting the performance of a heat exchanger it is necessary that the total heat transfer may be related with its governing parameters : (*i*) U (overall heat transfer coefficient) due to various modes of heat transfer, (*ii*) A total surface area of the heat transfer, and (*iii*) t_1, t_2 (the inlet and outlet fluid temperatures). Fig. 1 shows the overall energy balance in a heat exchanger.

Let, $\dot{m}$ = Mass flow rate, kg/s,

c_p = Specific heat of fluid at constant pressure, J/kg°C,

t = Temperature of fluid, °C, and

Δt = Temperature drop or rise of a fluid across the heat exchanger.

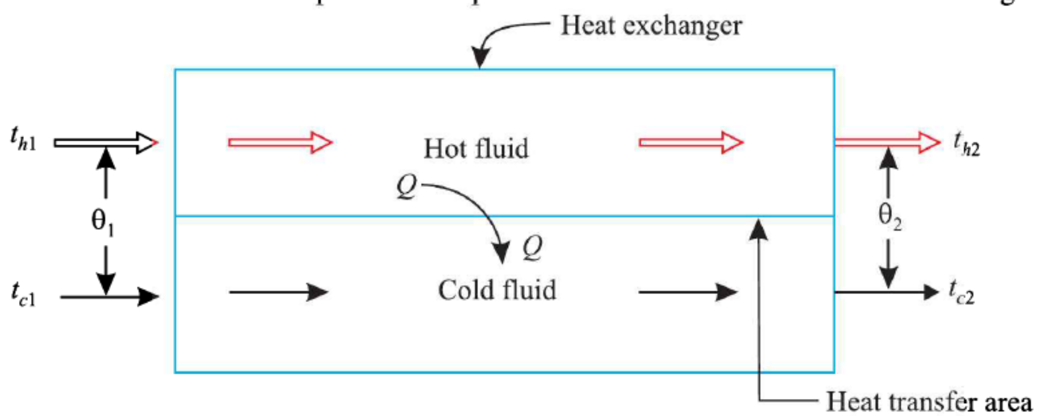


Fig. 1 Overall energy balance in a heat exchanger.

Subscripts *h* and *c* refer to the *hot* and *cold* fluids respectively; subscripts 1 and 2 correspond to the *inlet* and *outlet* conditions respectively.

Assuming that there is no heat loss to the surroundings and potential and kinetic energy changes are negligible, from the energy balance in a heat exchanger, we have :

Heat given up by the hot fluid, $Q = m_h \, c_{ph} \, (t_{h1} - t_{h2})$

Heat picked up by the cold fluid, $Q = m_c \, c_{pc} \, (t_{c2} - t_{c1})$

Total heat transfer rate in the heat exchanger, $Q = U A \theta_m$

where, U = Overall heat transfer coefficient between the two fluids,

A = Effective heat transfer area, and

θ_m = *Appropriate mean value of temperature difference or logarithmic mean temperature difference* (*LMTD*).

2- Logarithmic Mean Temperature Difference (LMTD)

Logarithmic mean temperature difference (LMTD) *is defined as* that *temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference.*

In order to derive expression for *LMTD* for various types of heat exchangers, the following **assumptions** are made :

1. The overall heat transfer coefficient U is constant.
2. The flow conditions are steady.
3. The specific heats and mass flow rates of both fluids are constant.
4. There is no loss of heat to the surroundings, due to the heat exchanger being perfectly insulated.
5. There is no change of phase either of the fluid during the heat transfer.
6. The changes in potential and kinetic energies are negligible.
7. Axial conduction along the tubes of the heat exchanger is negligible.

A- Logarithmic Mean Temperature Difference for ((Parallel Flow))

Refer to Fig.2, which shows the flow arrangement and distribution of temperature in a single-pass parallel flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U\, dA\, (t_h - t_c) = U \cdot dA \cdot \Delta t$$

As a result of heat transfer dQ through the area dA, the hot fluid is cooled by dt_h whereas the cold fluid is heated up by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\dot{m}_h \cdot c_{ph} \cdot dt_h = \dot{m}_c \cdot c_{pc} \cdot dt_c = U \cdot dA \cdot (t_h - t_c)$$

(Here d_{th} is – ve and d_{tc} is + ve)

or, $$dt_h = -\frac{dQ}{\dot{m}_h\, c_{ph}} = -\frac{dQ}{C_h}$$

and, $$dt_c = \frac{dQ}{\dot{m}_c\, c_{pc}} = \frac{dQ}{C_c}$$

where, $C_h = \dot{m}_h\ c_{ph}$ = Heat capacity or water equivalent of hot fluid, and

$C_c = \dot{m}_c\ c_{pc}$ = Heat capacity or water equivalent of cold fluid.

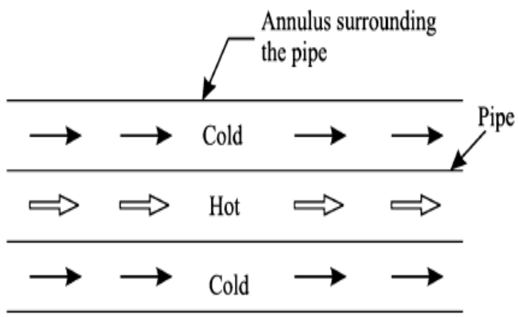
$\dot{m}_h$ and $\dot{m}_c$ are the mass flow rates of fluids and c_{ph} and c_{pc} are the respective specific heats.

$$\therefore \qquad dt_h - dt_c = -\,dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

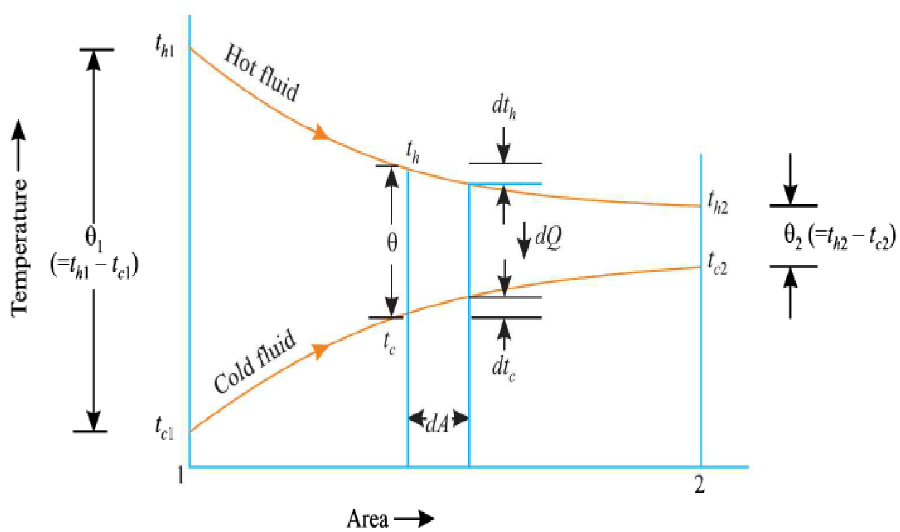
$$d\theta = -\,dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Substituting the value of dQ from the above equation becomes

$$d\theta = -\,U\,.\,dA\,(t_h - t_c)\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$



(*a*) Flow arrangement



(*b*) Temperature distribution

Subscripts h, c refer to : *hot* and *cold* fluids
Subscript 1, 2 refer to : *inlet* and *oulet* conditions.

Fig.2 Calculation of LMTD for a parallel flow heat exchanger.

or, $$d\theta = -U \cdot dA \cdot \theta \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or, $$\frac{d\theta}{\theta} = -U \cdot dA \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Integrating between inlet and outlet conditions (*i.e.* from $A = 0$ to $A = A$), we get

$$\int_1^2 \frac{d\theta}{\theta} = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \int_{A=0}^{A=A} U \cdot dA$$

or, $$ln\ (\theta_2/\theta_1) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Now, the total heat transfer rate between the two fluids is given by

$$Q = C_h (t_{h1} - t_{h2}) = C_c (t_{c2} - t_{c1})$$

or, $$\frac{1}{C_h} = \frac{t_{h1} - t_{h2}}{Q}$$

$$\frac{1}{C_c} = \frac{t_{c2} - t_{c1}}{Q}$$

Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ In above equation

$$ln\ (\theta_2/\theta_1) = -UA\left[\frac{t_{h1} - t_{h2}}{Q} + \frac{t_{c2} - t_{c1}}{Q}\right]$$

$$= \frac{UA}{Q}[(t_{h2} - t_{c2}) - (t_{h1} - t_{c1})] = \frac{UA}{Q}(\theta_2 - \theta_1)$$

$$Q = \frac{UA\ (\theta_2 - \theta_1)}{ln\ (\theta_2/\theta_1)}$$

The above equation may be written as

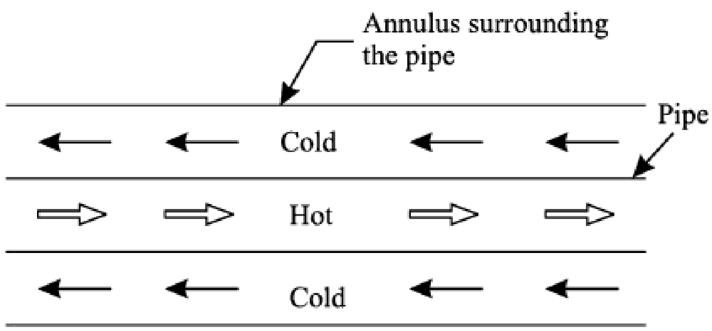
$$Q = U A \theta_m$$

where, $$\theta_m = \frac{\theta_2 - \theta_1}{ln\ (\theta_2/\theta_1)} = \frac{\theta_1 - \theta_2}{ln\ (\theta_1/\theta_2)}$$

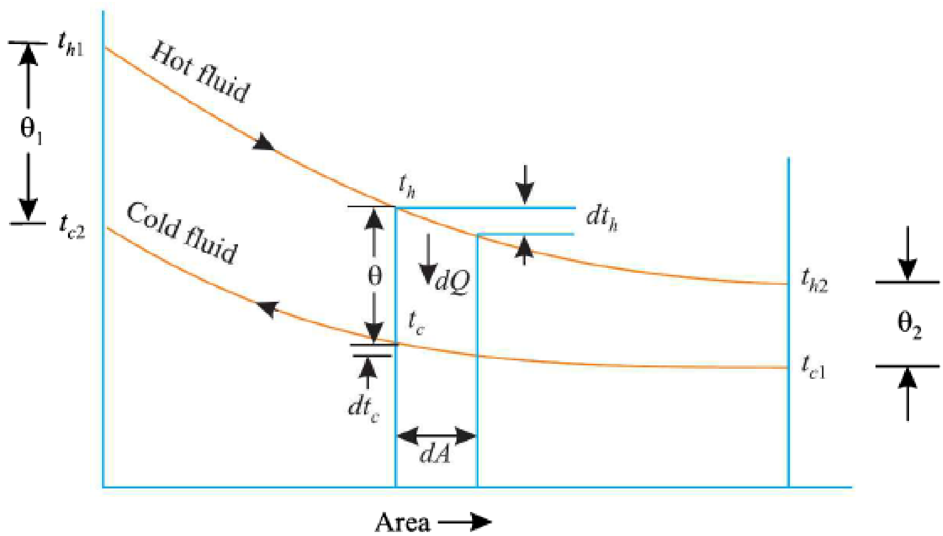
θ_m is called the *logarithmic mean temperature difference* (*LMTD*).

B- Logarithmic Mean Temperature Difference for ((Counter - Flow))

Refer to Fig.3, which shows the flow arrangement and temperature distribution in a single-pass counter-flow heat exchanger.



(*a*) Flow arrangement



(*b*) Temperature distribution

Fig.3 Calculation of LMTD for a counter-flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U \cdot dA\,(t_h - t_c) = U \cdot dA \cdot \Delta t$$

In this case also, due to heat transfer dQ through the area dA, the hot fluid is cooled down by dt_h whereas the cold fluid is heated by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\dot{m}_h \cdot c_{ph} \cdot dt_h = -\dot{m}_c \cdot c_{pc} \cdot dt_c$$

In a counter-flow system, the temperatures of both the fluids *decrease* in the direction of heat exchanger length, hence the – ve signs.

$\therefore$ $$dt_h = -\frac{dQ}{\dot{m}_h\, c_{ph}} = -\frac{dQ}{C_h}$$

and, $$dt_c = -\frac{dQ}{\dot{m}_c\, c_{pc}} = -\frac{dQ}{C_c}$$

$\therefore$ $$dt_h - dt_c = -\,dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or, $$d\theta = -\,dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

Inserting the value of dQ from above equation

$$d\theta = -\,U\;dA\,(t_h - t_c)\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$= -\,U\,dA\,.\,\theta\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or, $$\frac{d\theta}{\theta} = -\,U\,dA\,.\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

Integrating the above equation from $A = 0$ to $A = A$, we get

$$ln\,(\theta_2/\theta_1) = -\,U\,.\,A\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

Now, the total heat transfer rate between the two fluids is given by

$$Q = C_h\,(t_{h1} - t_{h2}) = C_c\,(t_{c2} - t_{c1})$$

or, $$\frac{1}{C_h} = \frac{t_{h1} - t_{h2}}{Q}$$

or, $$\frac{1}{C_c} = \frac{t_{c2} - t_{c1}}{Q}$$

substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$

$$ln\,(\theta_2/\theta_1) = -\,U\,A\left[\frac{t_{h1} - t_{h2}}{Q} - \frac{t_{c2} - t_{c1}}{Q}\right]$$

$$= -\frac{UA}{Q}\,[(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})] = -\frac{UA}{Q}\,(\theta_1 - \theta_2) = \frac{UA}{Q}\,(\theta_2 - \theta_1)$$

or, $$Q = \frac{UA\,(\theta_2 - \theta_1)}{ln\,(\theta_2/\theta_1)}$$

Since, $$Q = U\,A\,\theta_m$$

$$\therefore \qquad \theta_m = \frac{\theta_2 - \theta_1}{\ln(\theta_2/\theta_1)} = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)}$$

θ_m (*LMTD*) *for a counter-flow unit is always greater than that for a parallel flow unit*; hence counter-flow heat exchanger can transfer *more* heat than parallel-flow one; in other words a counter-flow heat exchanger needs a *smaller heating surface for the same rate of heat transfer*. For this reason, the *counter-flow arrangement is usually used*.

When the temperature variations of the fluids are relatively small, then temperature variation curves are approximately straight lines and adequately accurate results are obtained by taking the *arithmatic mean temperature difference* (*AMTD*).

$$AMTD = \frac{t_{h1} + t_{h2}}{2} - \frac{t_{c1} + t_{c2}}{2} = \frac{(t_{h1} - tc_1) + (t_{h2} - tc_2)}{2} = \frac{\theta_1 + \theta_2}{2}$$

However, practical considerations suggest that the logarithmic mean temperature difference (θ_m) should be invariably used when $\frac{\theta_1}{\theta_2} > 1.7$.

<u>You can reads the Examples in Rajput (10.1 to 10.30) from page.605</u>