Lecture Four

Design Analysis Methods in HE

1- Introduction.

A heat exchanger can be designed by the *LMTD* (logarithmic mean temperature difference) when *inlet and outlet conditions are specified.* However, when the problem is to determine the inlet or exit temperatures for a particular heat exchanger, the analysis is performed more easily, by using a method based on effectiveness of the heat exchanger (concept first proposed by Nusselt) and number of transfer units (*NTU*).

2- Effectiveness Method.

The **heat exchanger effectiveness** (ε) is defined as the *ratio of actual heat transfer to the maximum possible heat transfer.* Thus

$$\varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{Q}{Q_{\max}}$$

The actual heat transfer rate Q can be determined by writing an energy balance over either side of the heat exchanger.

$$Q = \dot{m}_h\, c_{ph}\, (t_{h1} - t_{h2}) = \dot{m}_c\, c_{pc}\, (t_{c2} - t_{c1})$$

The product of mass flow rate and the specific heat, as a matter of convenience, is defined as the fluid capacity rate C :

$\dot{m}_h\, c_{ph} = C_h$ = Hot fluid capacity rate

$\dot{m}_c\, c_{pc} = C_c$ = Cold fluid capcity rate

C_{min} = The minimum fluid capacity rate (C_h or C_c)

C_{max} = The maximum fluid capacity rate (C_h or C_c).

The *maximum rate of heat transfer for parallel flow or counter-flow heat exchangers would occur if the outlet temperature of the fluid with smaller value of C_h or C_c i.e., C_{min} were to be equal to the inlet temperature of the other fluid.* The maximum possible temperature change can be achieved by *only one of fluids*, depending upon their heat capacity rates. This maximum change cannot be obtained by both the fluids except in the very special case of *equal heat capacity rates*. Thus :

$$Q_{max} = C_h\,(t_{h1} - t_{c1}) \text{ or } C_c\,(t_{h1} - t_{c1})$$

Q_{max} is the *minimum* of these two values, *i.e.*, Once the effectiveness is known, the heat transfer rate can be very easily calculated by using the equation

$$Q_{max} = C_{min}\,(t_{h1} - t_{c1})$$

$$\therefore \quad \varepsilon = \frac{C_h\,(t_{h1} - t_{h2})}{C_{min}\,(t_{h1} - t_{c1})} = \frac{C_c\,(t_{c2} - t_{c1})}{C_{min}\,(t_{h1} - t_{c1})}$$

$$Q = \varepsilon\, C_{min}\,(t_{h1} - t_{c1})$$

3- Number of Transfer Units (NTU) Method.

that effectiveness ε is a function of several variables and as such it is inconvenient to combine them in a graphical or tabular form. However, by compiling a non-dimensional grouping, ε can be expressed as a function of three non-dimensional parameters. This method is known as ***NTU method***. This method/approach *facilitates the comparison between the various types of heat exchangers* which may be used for a particular application. The effectiveness expresssions for the parallel flow and counter-flow cases can be derived as follows :

I. Effectiveness for the ((Parallel-Flow)) HE:

The heat exchange dQ through an area dA of the heat exchanger is given by

$$dQ = U.dA\,(t_h - t_c) \qquad \ldots(i)$$

$$= -\,\dot{m}.c_{ph}\,.dt_h = \dot{m}_c\,.c_{pc}\,.dt_c$$

$$= -\,C_h.dt_h = C_c.dt_c \qquad \ldots(ii)$$

From expression (ii), we have

$$dt_h = \frac{-\,dQ}{C_h} \quad \text{and} \quad dt_c = \frac{dQ}{C_c}$$

$$\therefore \quad d\,(t_h - t_c) = -\,dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Substituting the value of dQ from expression (i) and rearranging, we get

$$\frac{d\,(t_h - t_c)}{(t_h - t_c)} = -\,U\,.dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Upon integration, we get

$$ln\left[\frac{(t_{h2}-t_{c2})}{(t_{h1}-t_{c1})}\right] = -UA\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

$$ln\left[\frac{(t_{h2}-t_{c2})}{(t_{h1}-t_{c1})}\right] = -\frac{UA}{C_h}\left(1+\frac{C_h}{C_c}\right)$$

Equation -----------(A) $$\left(\frac{t_{h2}-t_{c2}}{t_{h1}-t_{c1}}\right) = \exp\left[-(UA/C_h)\{1+(C_h/C_c)\}\right]$$

we have the expressions for effectiveness

$$\varepsilon = \frac{C_h\ (t_{h1}-t_{h2})}{C_{min}\ (t_{h1}-t_{c1})} = \frac{C_c\ (t_{c2}-t_{c1})}{C_{min}\ (t_{h1}-t_{c1})}$$

$$t_{h2} = t_{h1} - \frac{\varepsilon\, C_{min}\ (t_{h1}-t_{c1})}{C_h}$$

$$t_{c2} = t_{c1} + \frac{\varepsilon\, C_{min}\ (t_{h1}-t_{c1})}{C_c}$$

Eliminating t_{h2} and t_{c2} from Eq.A with help of above equations

$$\frac{1}{(t_{h1}-t_{c1})}\left[(t_{h1}-t_{c1}) - \varepsilon\, C_{min}\ (t_{h1}-t_{c1})\left(\frac{1}{C_h}+\frac{1}{C_c}\right)\right] = \exp\left[-(UA/C_h)\{1+C_h/C_c\}\right]$$

or, $$1-\varepsilon\, C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right) = \exp\left[-(UA/C_h)\{1+C_h/C_c\}\right]$$

Eq--------------(B) $$\varepsilon = \frac{1-\exp\left[-(UA/C_h)\{1+C_h/C_c\}\right]}{C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}$$

If $C_c > C_h$ then $C_{min} = C_h$ and $C_{max} = C_c$, hence eqn. (B) becomes

Eq. --------------(C) $$\varepsilon = \frac{1 - \exp\,[-\,(UA/C_{min})\,\{1 + C_{min}/C_{max})\}]}{1 + (C_{min}/C_{max})}$$

If $C_c < C_h$ then $C_{min} = C_c$ and $C_{max} = C_h$, hence eqn. (B) becomes

Eq. --------------(D) $$\varepsilon = \frac{1 - \exp\,[-\,(UA/C_{max})\,\{1 + C_{max}/C_{min})\}]}{1 + (C_{min}/C_{max})}$$

By rearranging eqns. (C) and (D) we get a common equation

$$\varepsilon = \frac{1 - \exp\,[-\,(UA/C_{min})\,\{1 + C_{min}/C_{max})\}]}{1 + (C_{min}/C_{max})}$$

where C_{min} and C_{max} represent the smaller and larger of the two heat capacities C_c and C_h.

- The grouping of the terms $(UA)/C_{min}$ is a dimensionless expression called the number of transfer units NTU; NTU is a *measure of effectiveness of the heat exchanger*.
- C_{min}/C_{max} is the second dimensionless parameter and is called the *capacity ratio R*.
- The last dimensionless parameter is the *flow arrangement*, *i.e.*, parallel flow, counter-flow, cross-flow and so on.

Thus the effectiveness of a parallel flow heat exchanger is given by

$$\varepsilon = \frac{1 - \exp\,[-\,NTU\;\{1 + (C_{min}/C_{max})\}]}{1 + (C_{min}/C_{max})}$$

or,
$$\varepsilon = \frac{1 - \exp\,[-\,NTU\;(1 + R)]}{1 + R}$$

II. Effectiveness for the ((Counter-Flow)) HE:

The heat exchange dQ through an area dA of the heat exchanger is given by

$$dQ = U.dA\,(t_h - t_c) \qquad ...(i)$$
$$= -\,\dot{m}\;c_{ph}\,dt_h = -\,\dot{m}\;c_{pc}\,dt_c$$
$$= -\,C_h\,dt_h = -\,C_c\,dt_c \qquad ...(ii)$$

From expression (ii), we have

$$dt_h = -\,\frac{dQ}{C_h} \text{ and } dt_c = -\,\frac{dQ}{C_c}$$

$$\therefore \quad d\,(t_h - t_c) = -\,dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] = dQ\left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

Substituting the value of dQ from expression (i), we get,

$$\frac{d\,(t_h - t_c)}{t_h - t_c} = U\,dA\left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

Upon integration, we get

$$ln\left[\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}}\right] = UA\left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

or,
$$ln\left[\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}}\right] = \frac{UA}{C_c}\left[1 - \frac{C_c}{C_h}\right]$$

Equation -----------(E)
$$\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}]$$

we have the expressions for effectiveness,

$$\varepsilon = \frac{C_h\,(t_{h1} - t_{h2})}{C_{min}\,(t_{h1} - t_{c1})} = \frac{C_c\,(t_{c2} - t_{c1})}{C_{min}\,(t_{h1} - t_{c1})}$$

Hence,
$$t_{h2} = t_{h1} - \frac{\varepsilon\,C_{min}\,(t_{h1} - t_{c1})}{C_h} \qquad ...(iii)$$

$$t_{c2} = t_{c1} + \frac{\varepsilon\,C_{min}\,(t_{h1} - t_{c1})}{C_c} \qquad ...(iv)$$

Substituting these values in eqn. (E) we get,

$$\frac{\left[t_{h1} - \dfrac{\varepsilon\,C_{min}\,(t_{h1} - t_{c1})}{C_h}\right] - t_{c1}}{t_{h1} - \left[t_{c1} + \dfrac{\varepsilon\,C_{min}\,(t_{h1} - t_{c1})}{C_c}\right]} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}]$$

$$\frac{(t_{h1} - t_{c1})\left[1 - \dfrac{\varepsilon\,.\,C_{min}}{C_h}\right]}{(t_{h1} - t_{c1})\left[1 - \dfrac{\varepsilon\,.\,C_{min}}{C_c}\right]} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}]$$

Equation -----------(F)

$$\frac{1-\dfrac{\varepsilon\,.\,C_{min}}{C_h}}{1-\dfrac{\varepsilon\,.\,C_{min}}{C_c}} = \exp\,[(UA/C_c)\,\{1-(C_c/C_h)\}]$$

Assume $C_c < C_h$, $C_c = C_{min}$ and $C_h = C_{max}$. Substituting these values is eqn. (F) , we get,

$$\frac{1-\dfrac{\varepsilon\,.\,C_{min}}{C_{max}}}{1-\dfrac{\varepsilon\,.\,C_{min}}{C_{min}}} = \exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}]$$

or,

$$\frac{1-\dfrac{\varepsilon\,.\,C_{min}}{C_{max}}}{1-\varepsilon} = \exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}]$$

or, $$1-\frac{\varepsilon.C_{min}}{C_{max}} = \exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}] - \exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}]\,\varepsilon$$

or, $$1-\exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}] = \varepsilon\left[\frac{C_{min}}{C_{max}} - \exp\left\{(UA/C_{min})\,(1-C_{min}/C_{max})\right\}\right]$$

or, $$\varepsilon = \frac{1-\exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}]}{\dfrac{C_{min}}{C_{max}} - \exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}]}$$

$$= \frac{\exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}]-1}{\exp\,[(UA/C_{min})\,\{1-(C_{min}/C_{max})\}] - \dfrac{C_{min}}{C_{max}}}$$

$$\varepsilon = \frac{1-\exp\,[(-\,UA/C_{min})\,\{1-(C_{min}/C_{max})\}]}{1-\dfrac{C_{min}}{C_{max}}\exp\,[(-\,UA/C_{min})\,\{1-(C_{min}/C_{max})\}]}$$

Since $C_{min}/C_{max} = R$ and $UA/C_{min} = NTU$, therefore,

$$\varepsilon = \frac{1-\exp\,[-\,NTU\;(1-R)]}{1-R\,\exp\,[-\,NTU\;(1-R)]}$$

We find that effectiveness of parallel flow and counter-flow heat exchangers is given by the following expressions :

$$(\varepsilon)_{parallel\ flow} = \frac{1 - \exp\left[-NTU\ (1 + R)\right]}{1 + R} \qquad ...(1)$$

$$(\varepsilon)_{counter\ flow} = \frac{1 - \exp\left[-NTU\ (1 - R)\right]}{1 - R\exp\left[-NTU\ (1 - R)\right]} \qquad ...(2)$$

where $R = (C_{min}/C_{max})$

Let us discuss *two limiting cases* of eqns. (1) and (2)

Case I : When $R \simeq 0$ **...Condensers and evaporators (boilers)**

By using the above case, we arrive at the following common expression for *parallel flow as well as counter-flow* heat exchangers

$$\varepsilon = 1 - exp\,(-NTU)$$

References.

Rajput , paragraph 10.7; and Examples 10.33 to 10.53; pp 645-675

Holmane, paragraph 10.6