# Lecture Ten

# Radiation Exchange Between Black Surfaces

## 1- Introduction.

We now consider the problem of radiative exchange between two or more surfaces. This exchange depends strongly on the surface geometries and orientations, as well as on their radiative properties and temperatures. Initially, we assume that the surfaces are separated by a *nonparticipating medium*. Since such a medium neither emits, absorbs, nor scatters, it has no effect on the transfer of radiation between surfaces. A vacuum meets these requirements exactly, and most gases meet them to an excellent approximation.

Our first objective is to establish geometrical features of the radiation exchange problem by developing the notion of a ***view factor***. Our second objective is to develop procedures for predicting radiative exchange between surfaces that form an *enclosure*. We will limit our attention to surfaces that are assumed to be opaque, diffuse, and gray. We conclude our consideration of radiation exchange between surfaces by considering the effects of a *participating medium*, namely, an intervening gas that emits and absorbs radiation.

## 2- View Factors-Integral Method.

Let us consider heat exchange between elementary areas $dA_1$ and $dA_2$ of two black radiating bodies, separated by a non-absorbing medium, and having areas $A_1$ and $A_2$ and temperatures $T_1$ and $T_2$ respectively. The elementary areas are at distance ( r ) apart and the normal to these areas make angles ($\theta_1$ and $\theta_2$)

Let $d\omega_1$ be subtended at $dA_1$ by $dA_2$

Let $d\omega_2$ be subtended at $dA_2$ by $dA_1$

Then

$$d\omega_1 = \frac{dA_2 \cos \theta_2}{r^2}, \quad \text{and} \quad d\omega_2 = \frac{dA_1 \cos \theta_1}{r^2} \qquad (1)$$

The energy leaving $dA_1$ in the direction given by the angle per unit solid angle = $I_{b1}\, dA_1 \cos \theta_1$.

where, $Ib$ = Black body intensity, and

$dA_1 \cos \theta_1$ = Projection of $dA_1$ on the line between the centres.

The rate of radiant energy leaving $dA_1$ and striking on $dA_2$ is given by

$$dQ_{1-2} = I_{b1}\, dA_1 \cos \theta_1 \, . \, d\omega_1$$
$$= \frac{I_{b1} \cos \theta_1 \cos \theta_2 \, dA_1 \, dA_2}{r^2} \qquad (2)$$

This energy is absorbed by the elementary area $dA_2$, since both the surfaces are black. The quantity of energy radiated by $dA_2$ and absorbed by $dA_1$ is given by

$$dQ_{2-1} = \frac{I_{b2} \cos\theta_2 \cos\theta_1 \, dA_2 \, dA_1}{r^2} \qquad (3)$$

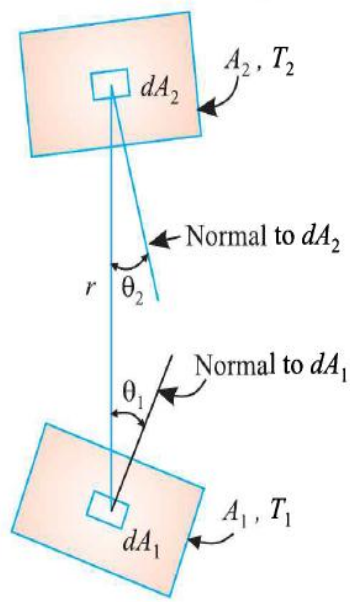


**Figure 1**: Radiation heat exchange between two black surfaces.

The net rate of transfer of energy between $dA_1$ and $dA_2$ is

$$dQ_{12} = dQ_{1-2} - dQ_{2-1}$$

$$= \frac{dA_1 \, dA_2 \cos\theta_1 \cos\theta_2}{r^2} (I_{b1} - I_{b2})$$

But, $$I_{b1} = \frac{E_{b1}}{\pi} \text{ and } I_{b2} = \frac{E_{b2}}{\pi}$$

$\therefore$ $$dQ_{12} = \frac{dA_1 \, dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2} (E_{b1} - E_{b2}) \qquad (4)$$

or, $$dQ_{12} = \frac{\sigma \, dA_1 \, dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2} (T_1^4 - T_2^4) \qquad (5)$$

The rate of total net heat transfer for the total areas $A_1$ and $A_2$ is given by

$$Q_{12} = \int dQ_{12} = \sigma (T_1^4 - T_2^4) \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{r^2} \qquad (6)$$

The rate of radiant energy emitted by $A_1$ that falls on $A_2$, from eqn. ( 2 ) is given by

$$Q_{1-2} = I_{b1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{r^2}$$

$$Q_{1-2} = \sigma T_1^4 \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2} \qquad (7)$$

The rate of total energy radiated by $A_1$ is given by

$$Q_1 = A_1 \sigma T_1^4$$

Hence the fraction of the rate of energy leaving area $A_1$ and impinging on area $A_2$ is given by

$$\frac{Q_{1-2}}{Q_1} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2} \qquad (8)$$

or, $$\frac{Q_{1-2}}{Q_1} = F_{1-2} \qquad (8.a)$$

$F_{1-2}$ is known as '**configuration factor**' or '**surface factor**' or '**view factor**' between the two radiating surfaces and is a function of geometry only.

Thus, the ***shape factor*** *may be defined as "The fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections."*

Further, $$Q_{1-2} = F_{1-2} A_1 \sigma T_1^4 \qquad (9)$$

Similarly, the rate of radiant energy by $A_2$ that falls on $A_1$, from eqn. ( 3 ) is given by

$$Q_{2-1} = \sigma T_2^4 \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2}$$

The rate of total energy radiated by $A_2$ is given by

$$Q_2 = A_2 \sigma T_2^4$$

Hence the fraction of the rate of energy leaving area $A_2$ and impinging on area $A_1$ is given by

$$\frac{Q_{2-1}}{Q_2} = \frac{1}{A_2} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2} \qquad (10)$$

or, $$\frac{Q_{2-1}}{Q_2} = F_{2-1}$$

$F_{2-1}$ is the shape factor of $A_2$ with respect to $A_1$.

$$Q_{2-1} = F_{2-1} A_2 \sigma T_2^4 \qquad (11)$$

From eqns. (12.8) and (12.10), we get

$$A_1 F_{1-2} = A_2 F_{2-1} \qquad (12)$$

The above result is known as **reciprocity theorem.** It indicates that the net radiant interchange may be evaluated by computing one way configuration factor from either surface to the other. Thus the net rate of heat transfer between two surfaces $A_1$ and $A_2$ is given by

$$Q_{12} = A_1 F_{1-2} \sigma (T_1^4 - T_2^4) = A_2 F_{2-1} \sigma (T_1^4 - T_2^4) \qquad (13)$$

It may be noted that eqn ( 13 ) is applicable to *black surfaces only and must not be used for surfaces having emissivities very different from unity.*

The evaluation of the integral equation 8 for determining the shape factor is rather complex and cumbersome. Therefore, results have been obtained and presented in graphical form for the geometries normally encountered in engineering practice. Geometrical factors for parallel planes (discs and rectangles) directly opposed and those for radiation between perpendicular rectangles with a common edge are shown in Figs. 2-3

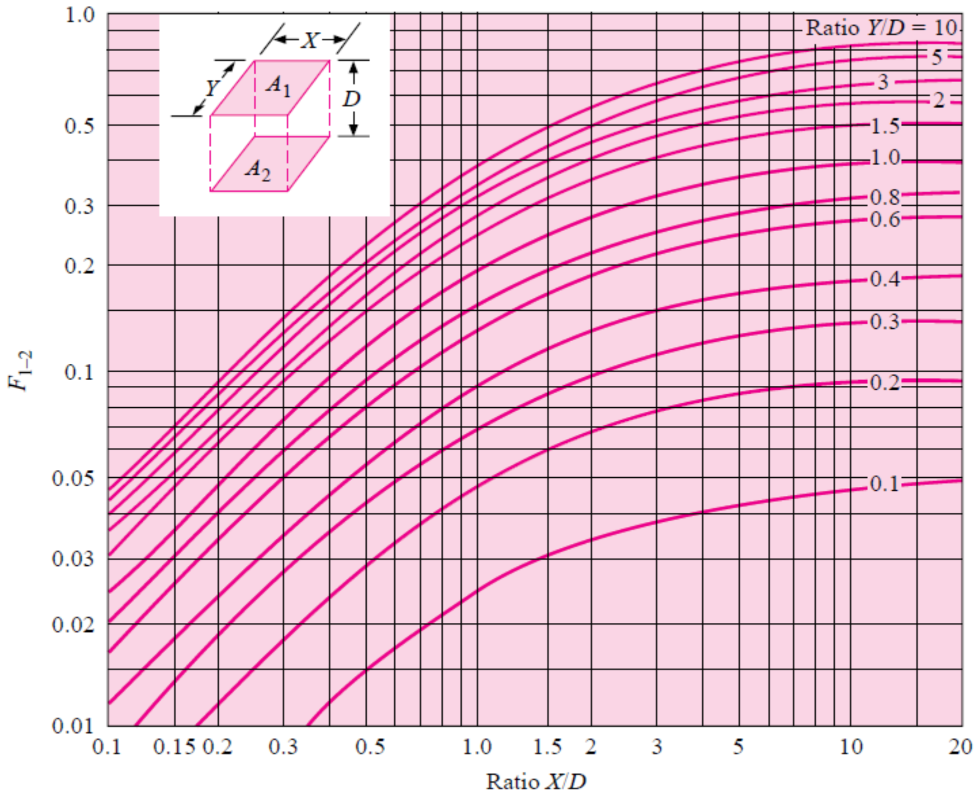


**Figure 2:** Shape factor for aligned parallel plates.

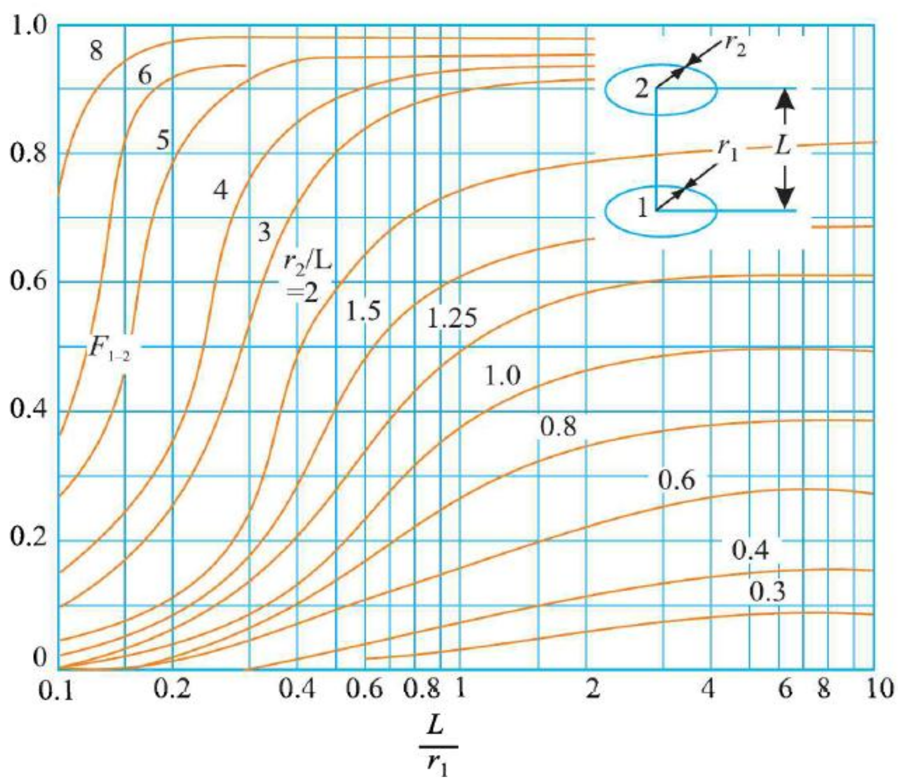


**Figure 3:** Shape factor for coaxial parallel discs.

**Ex-1.**

Two parallel black plates 0.5 by 1.0 m are spaced 0.5 m apart. One plate is maintained at 1000˚C and the other at 500˚C. What is the net radiant heat exchange between the two plates?

**Solution**

The ratios for use with Fig. (2) are

$$\frac{Y}{D} = \frac{0.5}{0.5} = 1.0 \qquad \frac{X}{D} = \frac{1.0}{0.5} = 2.0$$

so that $F_{12} = 0.285$. The heat transfer is calculated from

$$q = A_1 F_{12}(E_{b1} - E_{b2}) = \sigma A_1 F_{12}(T_1^4 - T_2^4)$$
$$= (5.669 \times 10^{-8})(0.5)(0.285)(1273^4 - 773^4)$$
$$= 18.33 \text{ kW} \quad [62,540 \text{ Btu/h}]$$