Lecture Eleven

Heat Exchange Between Nonblack Bodies

1- Introduction.

When nonblack bodies are involved, the energy striking a surface will not be absorbed; part will be reflected back to another heat-transfer surface, and part may be reflected out of the system entirely.

We shall assume that all surfaces considered in our analysis are diffuse, gray, and uniform in temperature and that the reflective and emissive properties are constant over all the surface. Two new terms may be defined:

G = irradiation
= total radiation incident upon a surface per unit time and per unit area

J = radiosity
= total radiation that leaves a surface per unit time and per unit area

2- Heat Exchange between two Surfaces

In addition to the assumptions stated above, we shall also assume that the radiosity and irradiation are uniform over each surface.

As shown in Fig.(1), the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted, or

$$J = \varepsilon * E_b + \rho * G \quad (1)$$

Where (ε) is the emissivity and (E_b) is the blackbody emissive power. Since the transmissivity is assumed to be zero, the reflectivity may be expressed as

$$\rho = 1-\alpha = 1-\varepsilon$$

So that

$$J = \varepsilon * E_b + (1-\varepsilon)G \quad (2)$$

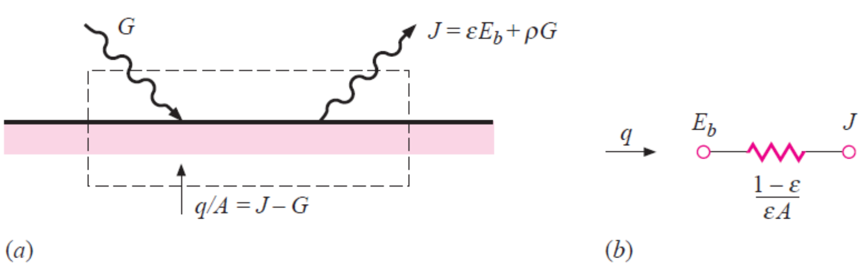


Figure 1: (*a*) Surface energy balance for opaque material; (*b*) element representing "surface resistance" in the radiation-network method.

The net energy leaving the surface is the difference between the radiosity and the irradiation:

$$\frac{q}{A} = J - G = \in E_b + (1 - \in)G - G$$

Solving for *G* in terms of *J* from Equation (2),

$$q = \frac{\epsilon A}{1-\epsilon}(E_b - J)$$

$$q = \frac{E_b - J}{(1-\epsilon)/\epsilon A} \qquad (3)$$

- In Eq. (3), If the denominator of the right side is considered as the surface resistance to radiation heat transfer.
- The numerator as a potential difference, and the heat flow as the "current," then a network element could be drawn as in Fig. (1-*b*) to represent the physical situation.
- Now consider the exchange of radiant energy by two surfaces, *A*1 and *A*2, shown in Fig. (2). Of that total radiation leaving surface 1, the amount that reaches surface 2 is $J_1A_1F_{12}$.
- The total energy leaving surface 2, this amount that reaches surface 1 is $J_2A_2F_{21}$.

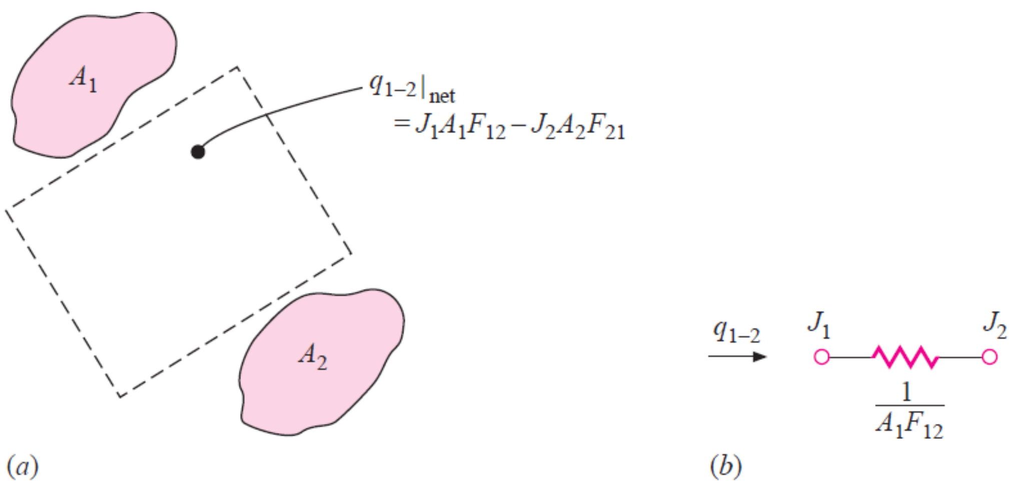


Figure 2: (*a*) Spatial energy exchange between two surfaces; (*b*) element representing "space resistance" in the radiation-network method.

The net interchange between the two surfaces is

$$q_{1-2} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

But

$$A_1 F_{12} = A_2 F_{21}$$

so that

$$q_{1-2} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21}$$

or

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}} \qquad (4)$$

Eq. (4) represent the essential of the radiation network method of two elements as shown in Fig's (1 and 2).

3- Network Radiation Heat Transfer Method.

To construct a network for a particular radiation heat-transfer problem for two surfaces we need

- Connect a "surface resistance" $\left[\frac{(1-\epsilon)}{\epsilon A}\right]$ to each surface.
- A "space resistance" $1/A_iF_{ij}$ between the radiosity potentials.

For example, two surfaces that exchange heat with each other *and nothing else* would be represented by the network shown in Fig.(3). In this case the net heat transfer would be the overall potential difference divided by the sum of the resistances:

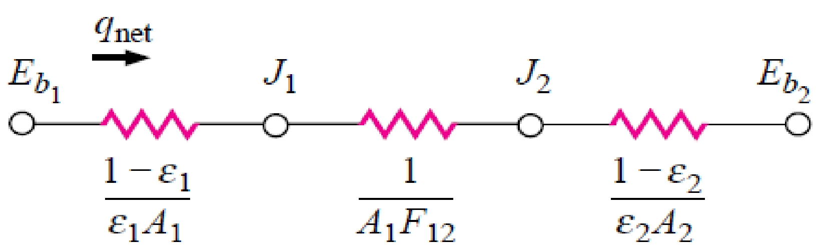


Figure 3: Radiation network for two surfaces that see each other and nothing else.

$$q_{net} = \frac{E_{b1} - E_{b2}}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2}$$
$$= \frac{\sigma(T_1^4 - T_2^4)}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2} \qquad (5)$$

A network for a three-body problem is shown in Fig. (4). In this case each of the bodies exchanges heat with the other two. The heat exchange between body 1 and body 2 would be

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$

and that between body 1 and body 3,

$$q_{1-3} = \frac{J_1 - J_3}{1/A_1 F_{13}}$$

To determine the heat flows in a problem of this type, the values of the radiosities must be calculated. This may be accomplished by performing standard methods of analysis used in dc circuit theory. The most convenient method is an application of Kirchhoff's current law to the circuit, which states that the sum of the currents entering a node is zero. **Example- 1** illustrates the use of the method for the three-body problem.

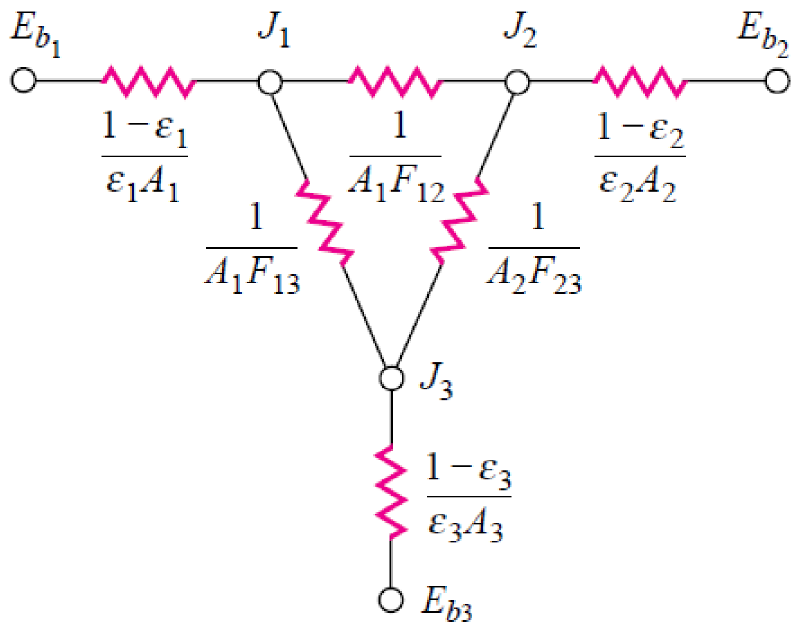
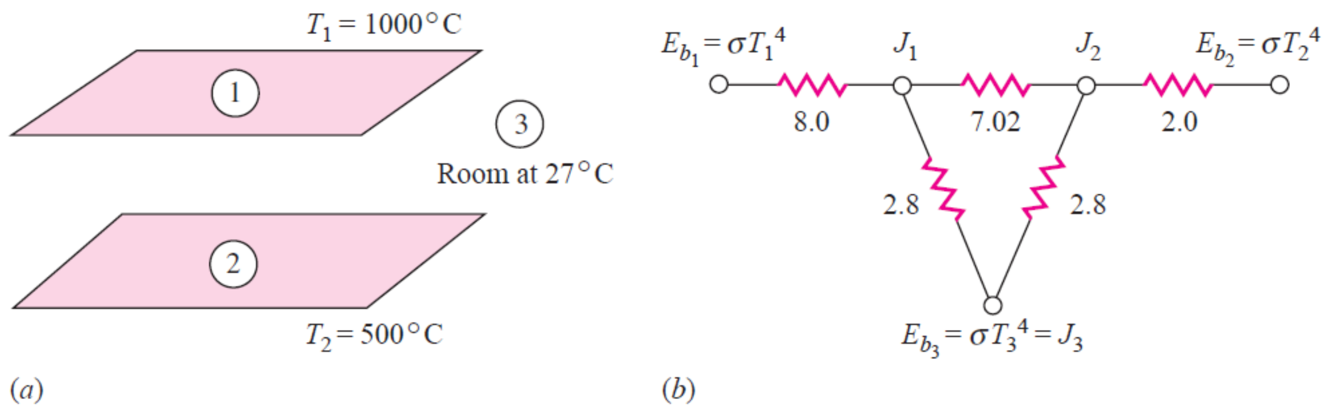


Figure 4: Radiation network for three surfaces that see each other and nothing else.

Ex.1

Two parallel plates 0.5 by 1.0 m are spaced 0.5 m apart, as shown in Figure. One plate is maintained at 1000˚C and the other at 500˚C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27˚C. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net transfer to each plate and to the room.



Solution

This is a three-body problem, the two plates and the room, so the radiation network is shown in above figure From the data of the problem

$$T_1 = 1000°C = 1273 \text{ K} \qquad A_1 = A_2 = 0.5 \text{ m}^2$$
$$T_2 = 500°C = 773 \text{ K} \qquad \epsilon_1 = 0.2$$
$$T_3 = 27°C = 300 \text{ K} \qquad \epsilon_2 = 0.5$$

Because the area of the room A_3 is very large, the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ may be taken as zero and we obtain $E_{b_3} = J_3$. The shape factor F_{12} was given in Ex-1 , L-10

$$F_{12} = 0.285 = F_{21}$$
$$F_{13} = 1 - F_{12} = 0.715$$
$$F_{23} = 1 - F_{21} = 0.715$$

The resistances in the network are calculated as

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.2}{(0.2)(0.5)} = 8.0 \qquad \frac{1-\epsilon_2}{\epsilon_2 A_2} = \frac{1-0.5}{(0.5)(0.5)} = 2.0$$
$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.5)(0.285)} = 7.018 \qquad \frac{1}{A_1 F_{13}} = \frac{1}{(0.5)(0.715)} = 2.797$$
$$\frac{1}{A_2 F_{23}} = \frac{1}{(0.5)(0.715)} = 2.797$$

Taking the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ as zero, we have the network as shown. To calculate the heat flows at each surface we must determine the radiosities J_1 and J_2. The network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero:

node J_1:

$$\frac{E_{b_1}-J_1}{8.0}+\frac{J_2-J_1}{7.018}+\frac{E_{b_3}-J_1}{2.797}=0 \qquad [a]$$

node J_2:

$$\frac{J_1-J_2}{7.018}+\frac{E_{b_3}-J_2}{2.797}+\frac{E_{b_2}-J_2}{2.0}=0 \qquad [b]$$

Now

$$E_{b_1}=\sigma T_1^4=148.87\ \text{kW/m}^2 \quad [47{,}190\ \text{Btu/h}\cdot\text{ft}^2]$$

$$E_{b_2}=\sigma T_2^4=20.241\ \text{kW/m}^2 \quad [6416\ \text{Btu/h}\cdot\text{ft}^2]$$

$$E_{b_3}=\sigma T_3^4=0.4592\ \text{kW/m}^2 \quad [145.6\ \text{Btu/h}\cdot\text{ft}^2]$$

Inserting the values of E_{b_1}, E_{b_2}, and E_{b_3} into Equations (a) and (b), we have two equations and two unknowns J_1 and J_2 that may be solved simultaneously to give

$$J_1=33.469\ \text{kW/m}^2 \qquad J_2=15.054\ \text{kW/m}^2$$

The total heat lost by plate 1 is

$$q_1=\frac{E_{b_1}-J_1}{(1-\epsilon_1)/\epsilon_1 A_1}=\frac{148.87-33.469}{8.0}=14.425\ \text{kW}$$

and the total heat lost by plate 2 is

$$q_2=\frac{E_{b_2}-J_2}{(1-\epsilon_2)/\epsilon_2 A_2}=\frac{20.241-15.054}{2.0}=2.594\ \text{kW}$$

The total heat received by the room is

$$q_3=\frac{J_1-J_3}{1/A_1F_{13}}+\frac{J_2-J_3}{1/A_2F_{23}}$$

$$=\frac{33.469-0.4592}{2.797}+\frac{15.054-0.4592}{2.797}=17.020\ \text{kW} \quad [58{,}070\ \text{Btu/h}]$$

From an overall-balance standpoint we must have