Introduction

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces.

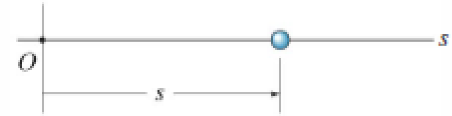
Engineering mechanics is divided into two areas of study, namely, statics and dynamics. Statics is concerned with the equilibrium of a body that is either at rest or moves with constant velocity.

dynamics, which deals with the accelerated motion of a body.

The subject of dynamics will be presented in two parts: kinematics, which treats only the geometric aspects of the motion, and kinetics, which is the analysis of the forces causing the motion.

Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

Position. *The straight-line path of a particle will be defined using a single coordinate axis s, Fig. 12-1a. The origin 0 on the path is a fixed point, and from this point the position coordinate s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from 0 to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s.*



Position

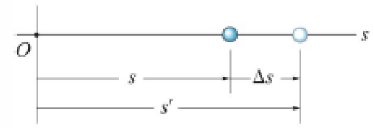
(a)

Displacement. The *displacement* of the particle is defined as the *change* in its *position*. For example, if the particle moves from one point to another, Fig. 12–1*b*, the displacement is

$$\Delta s = s' - s$$

In this case Δs is *positive* since the particle's final position is to the *right* of its initial position, i.e., $s' > s$. Likewise, if the final position were to the *left* of its initial position, Δs would be *negative*.

The displacement of a particle is also a *vector quantity*, and it should be distinguished from the distance the particle travels. Specifically, the *distance traveled* is a *positive scalar* that represents the total length of path over which the particle travels.



Displacement

(b)

Fig. 12–1

Velocity. If the particle moves through a displacement Δs during the time interval Δt, the *average velocity* of the particle during this time interval is

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt, the magnitude of Δs becomes smaller and smaller. Consequently, the *instantaneous velocity* is a vector defined as $v = \lim_{\Delta t \to 0} (\Delta s/\Delta t)$, or

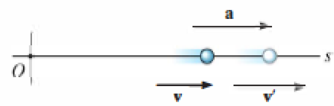
$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} \qquad (12\text{–}1)$$

Acceleration. Provided the velocity of the particle is known at two points, the *average acceleration* of the particle during the time interval Δt is defined as

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

Here Δv represents the difference in the velocity during the time interval Δt, i.e., $\Delta v = v' - v$, Fig. 12–1*e*.

The *instantaneous acceleration* at time t is a vector that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv, so that $a = \lim_{\Delta t \to 0} (\Delta v / \Delta t)$, or



Acceleration

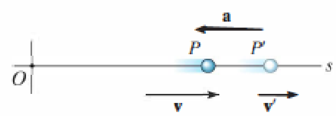
(e)

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} \qquad (12\text{–}2)$$

Substituting Eq. 12–1 into this result, we can also write

$$(\overset{+}{\rightarrow}) \qquad a = \frac{d^2 s}{dt^2}$$

Both the average and instantaneous acceleration can be either positive or negative. In particular, when the particle is *slowing down*, or its speed is decreasing, the particle is said to be *decelerating*. In this case, v' in Fig. 12–1*f* is *less* than v, and so $\Delta v = v' - v$ will be negative. Consequently, a will also be negative, and therefore it will act to the *left*, in the *opposite sense* to v. Also, note that when the *velocity* is *constant*, the *acceleration is zero* since $\Delta v = v - v = 0$. Units commonly used to express the magnitude of acceleration are m/s^2 or ft/s^2.



Deceleration

(f)

Finally, an important differential relation involving the displacement, velocity, and acceleration along the path may be obtained by eliminating the time differential dt between Eqs. 12–1 and 12–2, which gives

$$(\overset{+}{\rightarrow}) \qquad a\,ds = v\,dv \qquad (12\text{–}3)$$

Constant Acceleration, a = a$_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c\,ds = v\,dv$ can be integrated to obtain formulas that relate a_c, v, s, and t.

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_0^t a_c\,dt$$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t \qquad (12\text{–}4)$$

Constant Acceleration

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\, dt$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}5)$$

Velocity as a Function of Position. Either solve for t in Eq. 12–4 and substitute into Eq. 12–5, or integrate $v\, dv = a_c\, ds$, assuming that initially $v = v_0$ at $s = s_0$.

$$\int_{v_0}^{v} v\, dv = \int_{s_0}^{s} a_c\, ds$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} v^2 = v_0^2 + 2a_c(s - s_0) \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}6)$$

The algebraic signs of s_0, v_0, and a_c, used in the above three equations, are determined from the positive direction of the s axis as indicated by the arrow written at the left of each equation. Remember that these equations are useful *only when the acceleration is constant and when* $t = 0$, $s = s_0$, $v = v_0$. A typical example of constant accelerated motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the *downward* acceleration of the body when it is close to the earth is constant and approximately $9.81\ \text{m/s}^2$ or $32.2\ \text{ft/s}^2$. The proof of this is given in Example 13.2.

The car in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.

Fig. 12–2

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow \qquad \textit{Ans.}$$