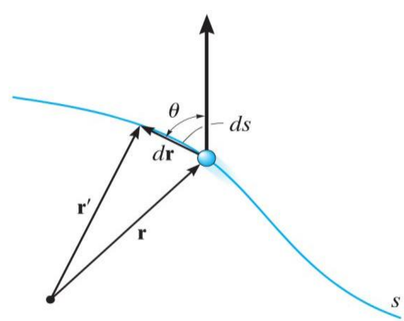
Kinetics of a Particle : Work and Energy

The Work of a Force



A force F will do work on a particle only when the particle undergoes a displacement in the direction of the force. For example, if the force F in Fig. 14-1 causes the particle to move along the path s from position r to a new position r', the displacement is then dr = r' - r. The magnitude of dr is ds, the length of the differential segment along the path. If the angle between the tails of dr and F is θ , Fig. 14-1 , then the work done by F is a scalar quantity, defined by

$dU = F\,ds\cos\theta$

Work of a Variable Force. If the particle acted upon by the force **F** undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ or s_1 to s_2, Fig. 14–2*a*, the work of force **F** is determined by integration. Provided **F** and θ can be expressed as a function of position, then

$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}\cdot d\mathbf{r} = \int_{s_1}^{s_2} F\cos\theta\, ds \qquad (14\text{–}1)$$

Sometimes, this relation may be obtained by using experimental data to plot a graph of $F\cos\theta$ vs. s. Then the *area* under this graph bounded by s_1 and s_2 represents the total work, Fig. 14–2*b*.

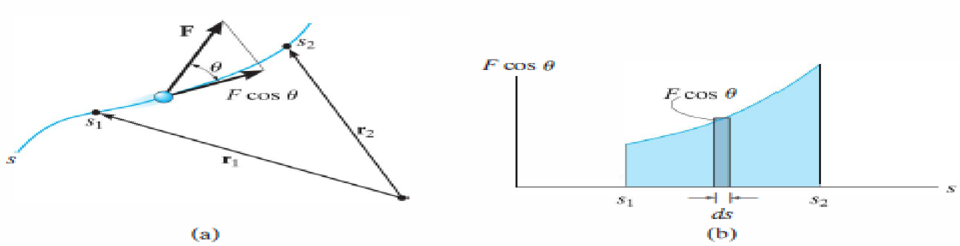
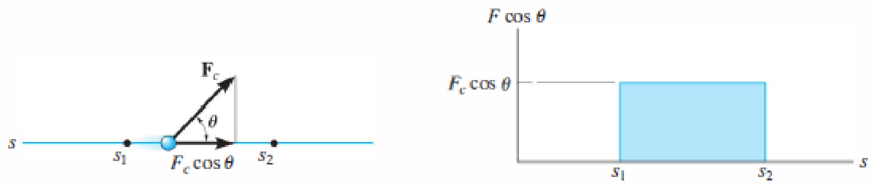


Fig. 14–2



(a) (b)

Fig. 14–3

Work of a Constant Force Moving Along a Straight Line.

If the force $\mathbf{F}_c$ has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig. 14–3*a*, then the component of $\mathbf{F}_c$ in the direction of displacement is always $F_c \cos \theta$. The work done by $\mathbf{F}_c$ when the particle is displaced from s_1 to s_2 is determined from Eq. 14–1, in which case

$$U_{1-2} = F_c \cos \theta \int_{s_1}^{s_2} ds$$

or

$$U_{1-2} = F_c \cos \theta (s_2 - s_1) \tag{14–2}$$

Here the work of $\mathbf{F}_c$ represents the *area of the rectangle* in Fig. 14–3*b*.

Work of a Weight.

Consider a particle of weight $\mathbf{W}$, which moves up along the path s shown in Fig. 14–4 from position s_1 to position s_2. At an intermediate point, the displacement $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$. Since $\mathbf{W} = -W\mathbf{j}$, applying Eq. 14–1 we have

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W \, dy = -W(y_2 - y_1)$$

or

$$U_{1-2} = -W \, \Delta y \tag{14–3}$$

Thus, the work is independent of the path and is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig. 14–4 the work is *negative*, since W is downward and Δy is upward. Note, however, that if the particle is displaced *downward* $(-\Delta y)$, the work of the weight is *positive*. Why?

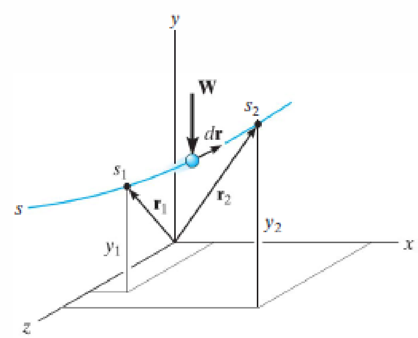


Fig. 14–4

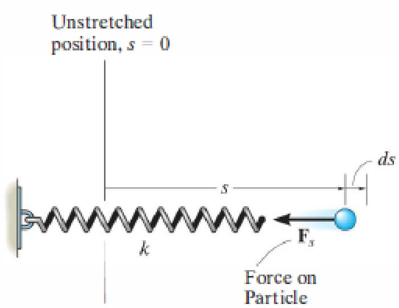
Work of a Spring Force. If an elastic spring is elongated a distance ds, Fig. 14–5*a*, then the work done by the force that acts on the attached particle is $dU = -F_s ds = -ks\,ds$. The work is *negative* since $\mathbf{F}_s$ acts in the opposite sense to ds. If the particle displaces from s_1 to s_2, the work of $\mathbf{F}_s$ is then

$$U_{1-2} = \int_{s_1}^{s_2} F_s\,ds = \int_{s_1}^{s_2} -ks\,ds$$

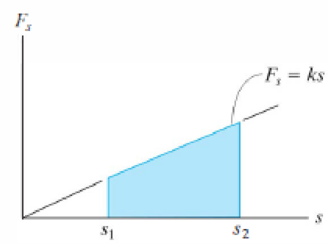
$$U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right) \qquad (14\text{–}4)$$

This work represents the trapezoidal area under the line $F_s = ks$, Fig. 14–5*b*.

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle—if both are in the *same sense, positive work* results; if they are *opposite* to one another, the *work is negative.*



(a)

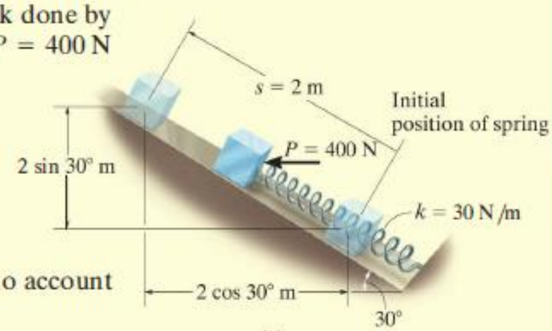


(b)

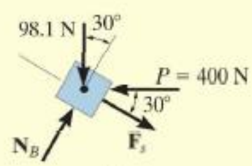
Fig. 14–5

EXAMPLE

The 10-kg block shown in Fig. 14–6*a* rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.



(a)



(b)

Fig. 14–6

SOLUTION

First the free-body diagram of the block is drawn in order to account for all the forces that act on the block, Fig. 14–6*b*.

Horizontal Force *P*. Since this force is *constant*, the work is determined using Eq. 14–2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e.,

$$U_P = 400\text{ N }(2\text{ m}\cos 30^\circ) = 692.8\text{ J}$$

or the displacement times the component of force in the direction of displacement, i.e.,

$$U_P = 400\text{ N}\cos 30^\circ(2\text{ m}) = 692.8\text{ J}$$

Spring Force F_s. In the initial position the spring is stretched $s_1 = 0.5$ m and in the final position it is stretched $s_2 = 0.5\text{ m} + 2\text{ m} = 2.5$ m. We require the work to be negative since the force and displacement are opposite to each other. The work of $\mathbf{F}_s$ is thus

$$U_s = -\left[\tfrac{1}{2}(30\text{ N/m})(2.5\text{ m})^2 - \tfrac{1}{2}(30\text{ N/m})(0.5\text{ m})^2\right] = -90\text{ J}$$

Weight *W*. Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1\text{ N})\,(2\text{ m}\sin 30^\circ) = -98.1\text{ J}$$

Note that it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1\sin 30^\circ\text{ N})\,(2\text{ m}) = -98.1\text{ J}$$

Normal Force N_B. This force does *no work* since it is *always* perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced 2 m is therefore

$$U_T = 692.8\text{ J} - 90\text{ J} - 98.1\text{ J} = 505\text{ J} \qquad \textit{Ans.}$$