Newton's Second Law of Motion

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change.
The basis for kinetics is Newton's second law, which states that when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force.
Newton's second law of motion may be written in mathematical form as

$$F = ma$$

The Equation of Motion

When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces; i.e., $F_R = \sum R$. *For this more general case, the equation of motion may be written as*

$$\sum F = ma$$

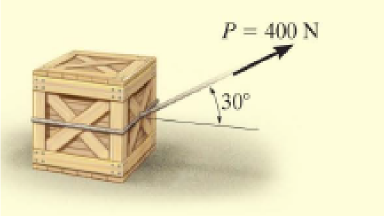
Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial x, y, Z frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their i, j, k components

$$\sum F = ma; \sum F_x i + \sum F_y j + \sum F_z k = m\left(a_x i + a_y j + a_z k\right)$$

EXAMPLE

EXAMPLE 1



(a)

The 50-kg crate shown in Fig. 13–6*a* rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

SOLUTION

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg = 50\text{ kg}\,(9.81\text{ m/s}^2) = 490.5\text{ N}$. As shown in Fig. 13–6*b*, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration **a** is assumed to act horizontally, in the positive *x* direction. There are two unknowns, namely N_C and a.

Equations of Motion. Using the data shown on the free-body diagram, we have

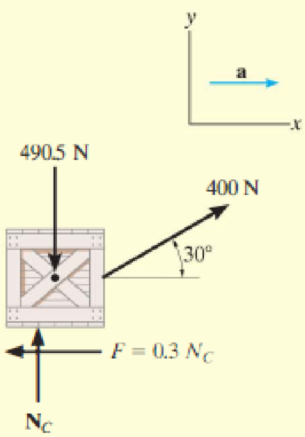
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 400\cos 30^\circ - 0.3N_C = 50a \qquad (1)$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400\sin 30^\circ = 0 \qquad (2)$$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving for a yields

$$N_C = 290.5\text{ N}$$

$$a = 5.185\text{ m/s}^2$$



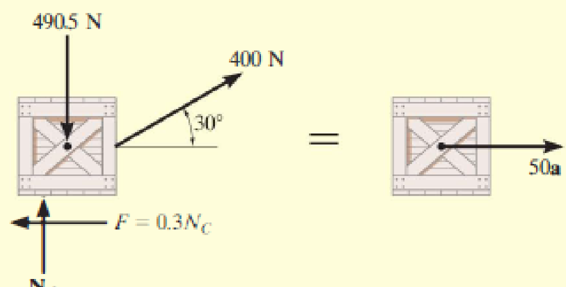
(b)

Fig. 13–6

Kinematics. Notice that the acceleration is *constant*, since the applied force **P** is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 5.185(3)$$

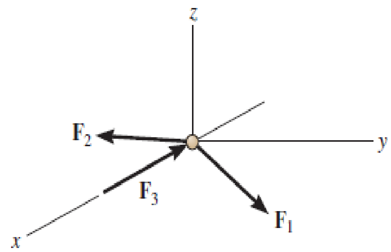
$$= 15.6\text{ m/s} \rightarrow \qquad \textit{Ans.}$$



(c)

EXA M P L E 2

The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



SOLUTION

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

Equating components:

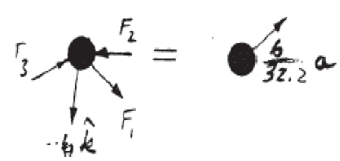
$$\left(\frac{6}{32.2}\right)a_x = t^2 - 2t + 2 \quad \left(\frac{6}{32.2}\right)a_y = -4t + 6 \quad \left(\frac{6}{32.2}\right)a_z = -2t - 7$$

Since $dv = a\,dt$, integrating from $v = 0, t = 0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3} - t^2 + 2t \quad \left(\frac{6}{32.2}\right)v_y = -2t^2 + 6t \quad \left(\frac{6}{32.2}\right)v_z = -t^2 - 7t$$

Since $ds = v\,dt$, integrating from $s = 0, t = 0$ yields

$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 \quad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3} + 3t^2 \quad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3} - \frac{7t^2}{2}$$



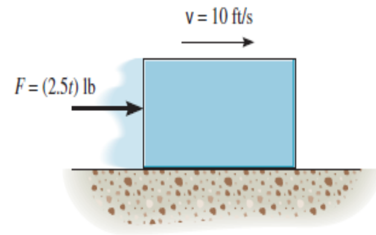
When $t = 2$ s then, $s_x = 14.31$ ft, $s_y = 35.78$ ft $s_z = -89.44$ ft

Thus,

$$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4 \text{ ft}$$ **Ans.**

Example 3

The 10-lb block has an initial velocity of 10 ft/s on the smooth plane. If a force $F = (2.5t)$ lb, where t is in seconds, acts on the block for 3 s, determine the final velocity of the block and the distance the block travels during this time.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 2.5t = \left(\dfrac{10}{32.2}\right)a$

$$a = 8.05t$$

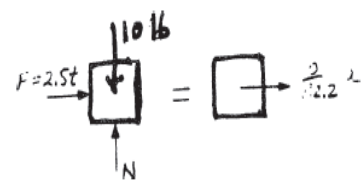
$$dv = a\,dt$$

$$\int_{10}^{v} dv = \int_0^t 8.05t\,dt$$

$$v = 4.025t^2 + 10$$

When $t = 3$ s,

$v = 46.2$ ft/s **Ans.**



$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (4.025t^2 + 10)\,dt$$

$$s = 1.3417t^3 + 10t$$

When $t = 3\,s$,

$s = 66.2$ ft **Ans.**