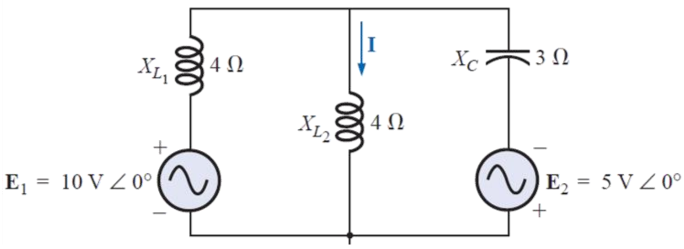
Network Theorems (ac)

SUPERPOSITION THEOREM

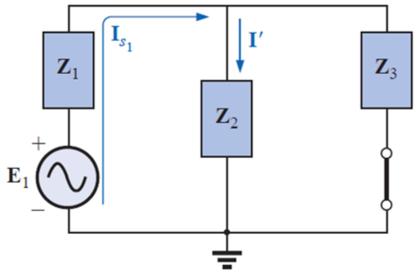
One of the most frequent applications of the superposition theorem is to electronic systems in which the dc and ac analyses are treated separately and the total solution is the sum of the two. It is an important application of the theorem because the impact of the reactive elements changes dramatically in response to the two types of independent sources.

Example:

Using the superposition theorem, find the current **I** through the 4Ω reactance (X_{L2})



Solution:



$$\mathbf{Z}_1 = +j X_{L_1} = j\,4\ \Omega$$
$$\mathbf{Z}_2 = +j X_{L_2} = j\,4\ \Omega$$
$$\mathbf{Z}_3 = -j X_C = -j\,3\ \Omega$$

Considering the effects of the voltage source $\mathbf{E}_1$

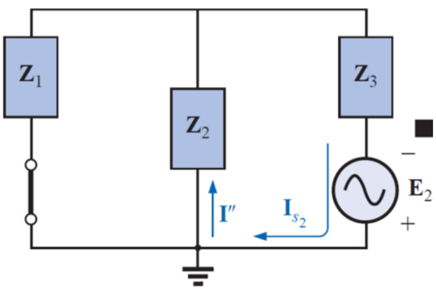
$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\ \Omega)(-j\,3\ \Omega)}{j\,4\ \Omega - j\,3\ \Omega} = \frac{12\ \Omega}{j} = -j\,12\ \Omega$$
$$= 12\ \Omega \angle -90°$$

$$I_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\ \text{V} \angle 0°}{-j\,12\ \Omega + j\,4\ \Omega} = \frac{10\ \text{V} \angle 0°}{8\ \Omega \angle -90°}$$
$$= 1.25\ \text{A} \angle 90°$$

and

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$
$$= \frac{(-j\,3\ \Omega)(j\,1.25\ \text{A})}{j\,4\ \Omega - j\,3\ \Omega} = \frac{3.75\ \text{A}}{j\,1} = 3.75\ \text{A} \angle -90°$$

Considering the effects of the voltage source $\mathbf{E}_2$, we have



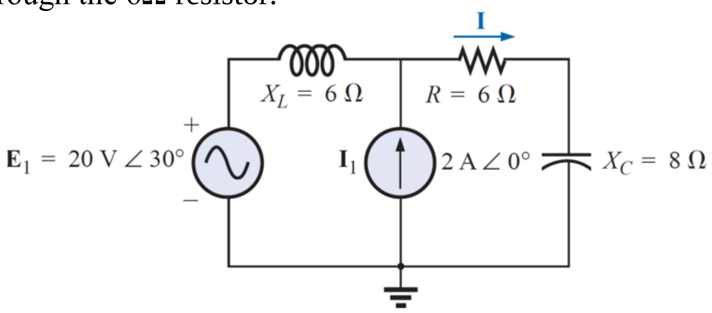
$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\ \Omega}{2} = j\,2\ \Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\ \text{V} \angle 0°}{j\,2\ \Omega - j\,3\ \Omega} = \frac{5\ \text{V} \angle 0°}{1\ \Omega \angle -90°} = 5\ \text{A} \angle 90°$$

$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\ \text{A} \angle 90°$$

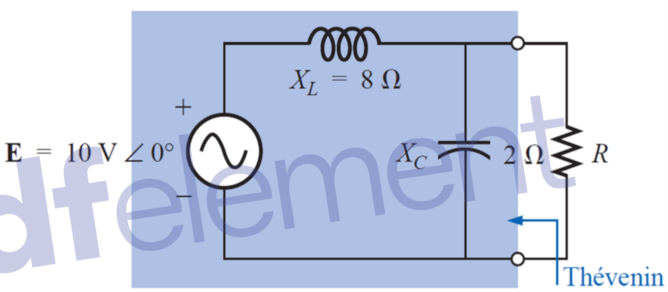
$$\mathbf{I} = \mathbf{I}' - \mathbf{I}''$$
$$= 3.75\ \text{A} \angle -90^\circ - 2.50\ \text{A} \angle 90^\circ = -j\,3.75\ \text{A} - j\,2.50\ \text{A}$$
$$= -j\,6.25\ \text{A}$$
$$\mathbf{I} = \mathbf{6.25\ A} \angle \mathbf{-90^\circ}$$

Example:
Using superposition, find the current **I** through the 6Ω resistor.



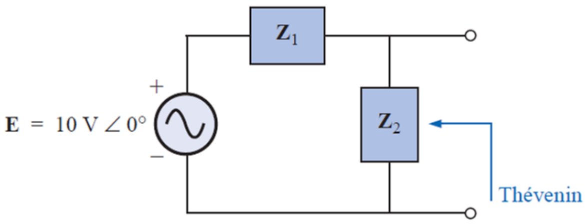
THE´VENIN’S THEOREM

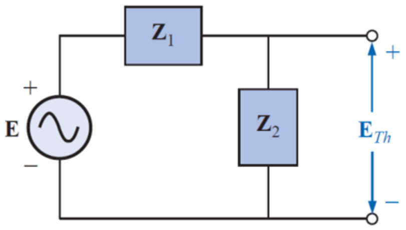
Example:
Find the Thévenin equivalent circuit for the network external to resistor *R*

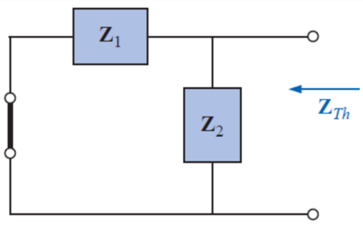


Solution:

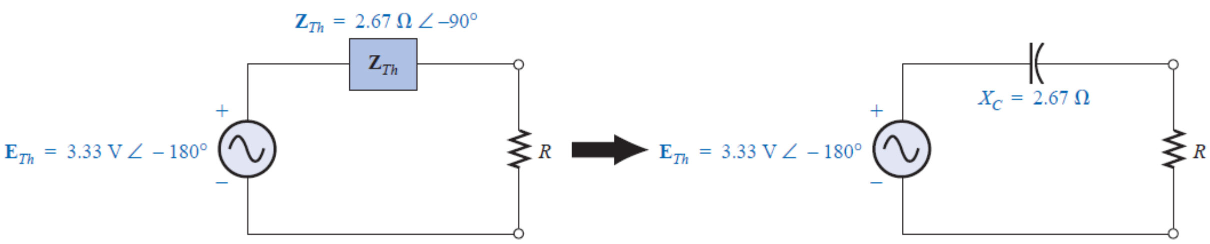
$$Z_1 = jX_L = j8\ \Omega$$
$$Z_2 = -jX_C = -j2\ \Omega$$
$$Z_{th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{16}{j6} = 2.67\angle -90\ \ \Omega$$
$$E_{th} = \frac{E Z_2}{Z_1 + Z_2} = \frac{-j20}{j6} = 3.33\angle -180\ V$$





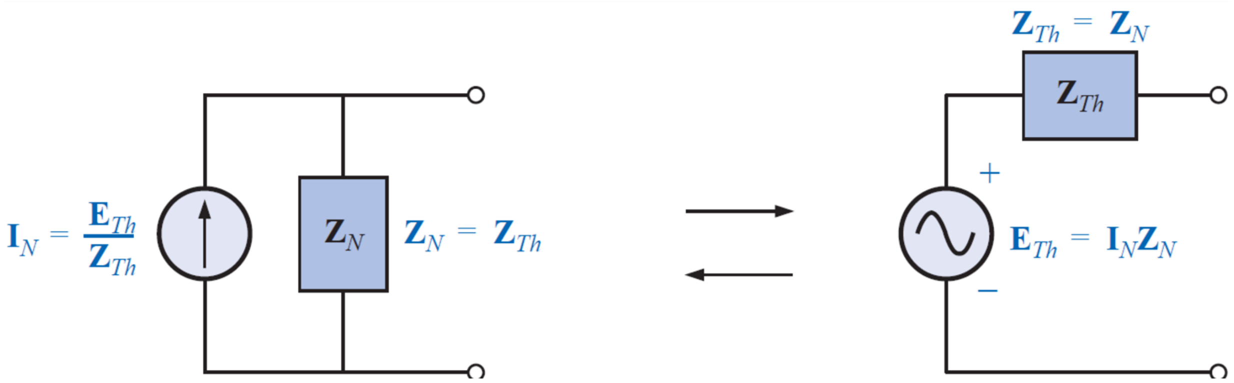


The Thévenin equivalent circuit is shown below



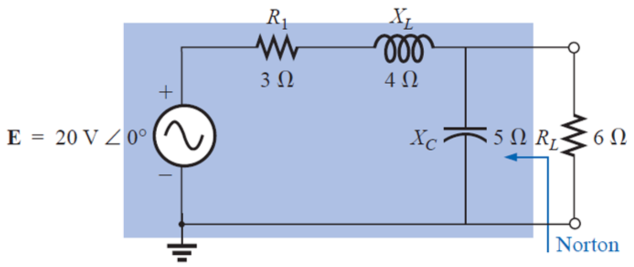
NORTON'S THEOREM

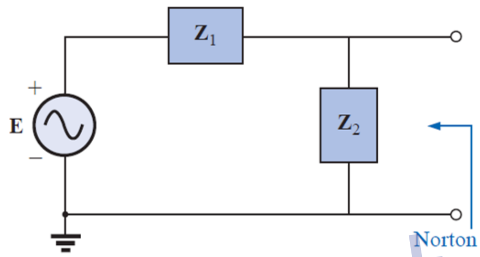
The Norton and Thévenin equivalent circuits can be found from each other by using the source transformation shown in figurer below.



Example:

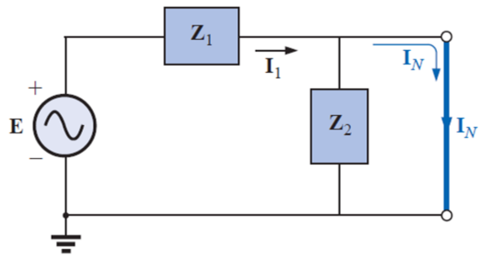
Determine the Norton equivalent circuit for the network external to the 6Ω resistor

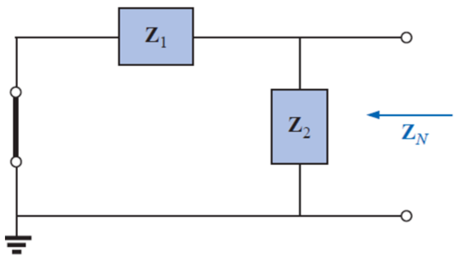




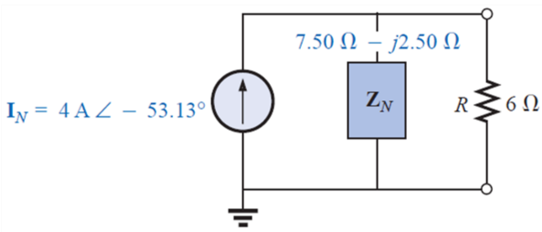
Solution:

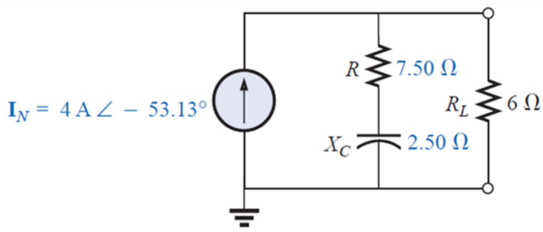
$$Z_1 = R_1 + jX_L = 3 + j4 = 5\angle 53.1\ \Omega$$
$$Z_2 = -jX_C = -j5\ \Omega$$
$$Z_N = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{5\angle 53.1 \times 5\angle -90}{3 + j4 - j5} = 7.91\angle -18.44 = \mathbf{7.50 - j\,2.50\Omega}$$
$$I_N = \frac{E}{Z_1} = \frac{20}{5\angle 53.1} = 4\angle -53.1\ A$$





The Norton equivalent circuit is shown in figure below





MAXIMUM POWER TRANSFER THEOREM

When applied to ac circuits, the **maximum power transfer theorem** states that ***maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.***

That is, for maximum power transfer to the load,

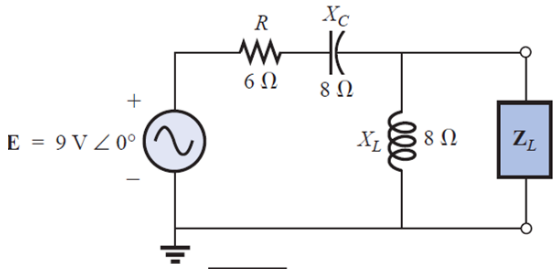
$Z_L = Z_{th}^* \qquad \theta_L = -\theta_{th}$

Therefore

$Z_T = R \mp jX + R \pm jX = 2R$

Example:

Find the load impedance for maximum power to the load, and find the maximum power.



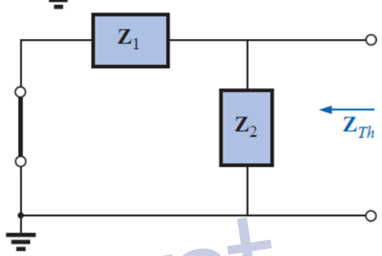
$Z_1 = 6 - j8 = 10\angle -53.1$

$Z_2 = j8$

$$Z_{th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{10\angle -53.1 \times 8\angle 90}{6 - j8 + j8} = 13.33\ \angle 36.87°$$

$= 10.66 + j8$

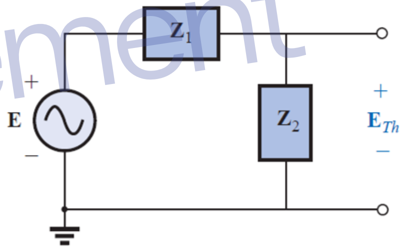
$Z_L = Z_{th}^* = 13.33\ \angle -36.87° = 10.66 - j8$



$$E_{th} = \frac{E Z_2}{Z_1 + Z_2} = \frac{9\angle 0 \times 8\angle 90}{6 - j8 + j8} = 12\ \angle 90°\ \text{V}$$

Then

$$P_{max} = \frac{E_{th}^2}{4R} = \frac{12^2}{4 \times 10.66} = 3.38\ w$$



Example:

Find the load impedance for maximum power to the load, and find the maximum power