Power (ac)

For any system, the power delivered to a load at any instant is defined by the product of the applied voltage and the resulting current; that is,

$$p = vi$$

In this case, since *v* and *i* are sinusoidal quantities, let us establish a general case where

$$v = V_m \sin(\omega t + \theta)$$
$$i = I_m \sin \omega t$$

Substituting the above equations for *v* and *i* into the power equation will result in

$$p = vi = V_m I_m \sin \omega t \sin(\omega t + \theta)$$
$$p = VI cos\theta(1 - cos2\omega t) + VI \sin\theta \sin 2\omega t$$

RESISTIVE CIRCUIT

For a purely resistive circuit , *v* and *i* are in phase,

$$p_R = VI cos0(1 - cos2\omega t) + VI \sin 0 \sin 2\omega t$$
$$p_R = VI(1 - cos2\omega t)$$

APPARENT POWER

$$S = VI \quad volt - amperes, VA$$
$$p = S cos\theta = SF_p$$

INDUCTIVE CIRCUIT AND REACTIVE POWER

For a purely inductive circuit , *v* leads *i* by 90°,

$$p_L = VI cos90(1 - cos2\omega t) + VI \sin 90 \sin 2\omega t$$
$$p_L = VI \sin 2\omega t$$

In general, the reactive power associated with any circuit is defined to be $VI \sin\theta$,
The symbol for reactive power is *Q*, and its unit of measure is the *volt-ampere reactive* (VAR).

$$Q = VI \sin\theta \quad \text{volt-ampere reactive, VAR}$$

For the inductor

$$Q_L = VI$$
$$F_p = cos\theta = cos90 = 0$$

CAPACITIVE CIRCUIT

$$p_C = VI cos(-90)(1 - cos2\omega t) + VI \sin(-90) \sin 2\omega t$$
$$p_C = -VI \sin 2\omega t$$
$$Q_C = VI \qquad \text{VAR}$$
$$F_p = cos\theta = cos90 = 0$$

THE POWER TRIANGLE

The three quantities **average power, apparent power,** and **reactive power** can be related in the vector domain by

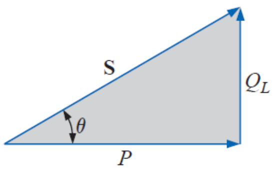
$$S = P + Q$$

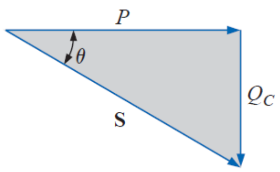
For an inductive load, the *phasor power* **S,** as it is often called, is defined by

$$S = P + jQ_L$$

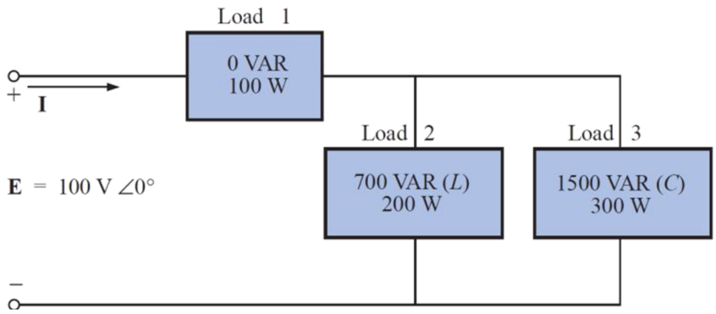
For a capacitive load, the phasor power **S** is defined by

$$S = P - jQ_C$$

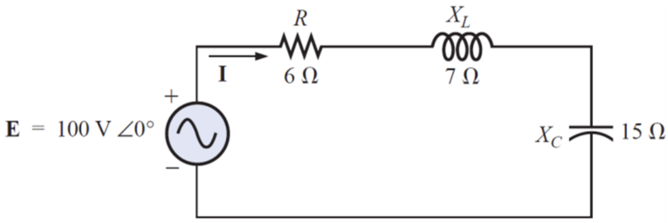




Example:
Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p of the network. Draw the power triangle and find the current in phasor form.



Example:
Find the total number of watts, volt-amperes reactive, and volt amperes, and the power factor F_p for the network and sketch the power triangle.



Example:
An electrical device is rated 5 kVA, 100 V at a 0.6 power-factor lag. What is the impedance of the device in rectangular coordinates?

POWER-FACTOR CORRECTION

The process of introducing reactive elements to bring the power factor closer to unity is called **power-factor correction.** Since most loads are inductive, the process normally involves introducing elements with capacitive terminal characteristics having the sole purpose of improving the power factor.

Example:
A 5-hp motor with a 0.6 lagging power factor and an efficiency of 92% is connected to a 208-V, 60-Hz supply.
a. Establish the power triangle for the load.
b. Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.
c. Determine the change in supply current from the uncompensated to the compensated system.

Example:
a. A small industrial plant has a 10-kW heating load and a 20-kVA inductive load due to a bank of induction motors. The heating elements are considered purely resistive ($Fp = 1$), and the induction motors have a lagging power factor of 0.7. If the supply is 1000 V at 60 Hz, determine the capacitive element required to raise the power factor to 0.95.
b. Compare the levels of current drawn from the supply.