III Hyperbolic Equation :

(i) Wave Equation: $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ $\quad 0 \leq x \leq l$

$t \geq 0$

$$\frac{1}{K^2}\left(U_{i,j-1} - 2U_{i,j} + U_{i,j+1}\right) = \frac{1}{h^2}\left(U_{i-1,j} - 2U_{i,j} + U_{i+1,j}\right)$$

where $x = i.h \quad (i = 0, 1, 2 \ldots)$

$t = j.K \quad (j = 0, 1, 2 \ldots)$

Let : $\alpha = \frac{k}{h}$ $\Rightarrow$

$$\boxed{U_{i,j+1} = \alpha^2\left(U_{i-1,j} + U_{i+1,j}\right) + 2\left(1-\alpha^2\right)\left(U_{i,j}\right) - U_{i,j-1}}$$

This equation is conditionally stable at $\alpha \leq 1.0$

Ex| The transverse displacement (u) of a point at a distance (x) from any end at any time (t) of a vibrating string satisfies the equation :

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 0 \leq x \leq 1 \quad , t \geq 0$$

With the B.C's :

$U = 0$ at $x = 0 , t \geq 0$

$U = 0$ at $x = 1 , t \geq 0$

With the I.C's :

$U = \frac{1}{2}x(1-x)$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0 , \quad 0 \leq x \leq 1$

Solve the above equation for $(0 \leq x \leq 1.0 , 0 \leq t \leq 0.4)$ with $h = k = 0.1$?

Initially the values will be as follows:

initially

$\leftarrow \uparrow \rightarrow$ symm.

	X	0	0.1	0.2	0.3	0.4	0.5	1.6	0.7	0.8	0.9	1.0
$j=0$	$u = \frac{1}{2}x(1-x)$	0	0.045	0.08	0.105	0.12	0.1	0.12	0.105	0.08	0.045	0

For other j's use the previous equations $j=0 \rightarrow j=4$