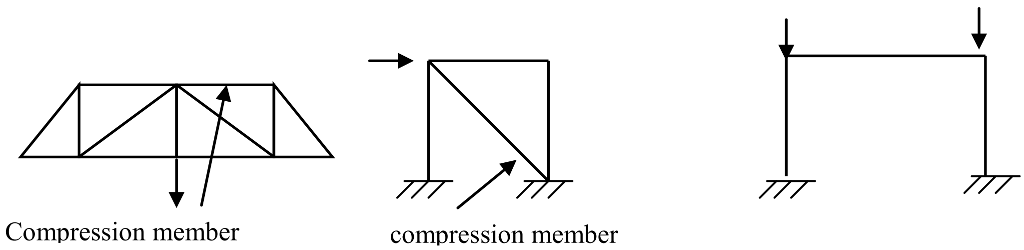
Chapter Three
Compression Members

1- Introduction

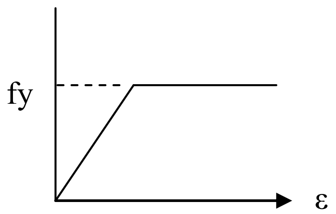
compression members are members subjected to direct axial compression force, like:



Sections used for column are the same for tension but especially pipes and double angle or double tee are widely used.

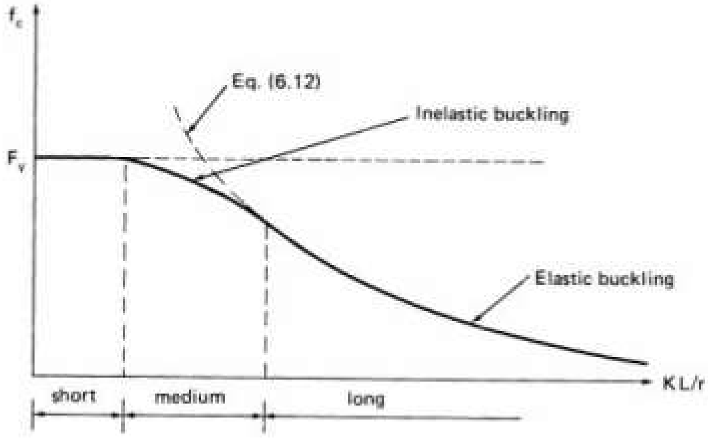
2 – Failure of column

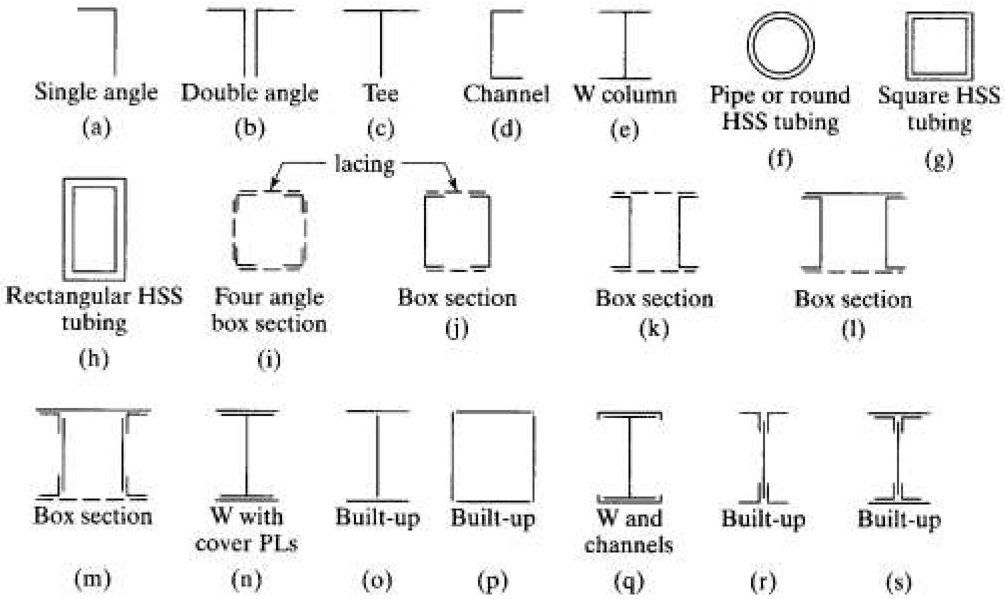
1- *Short columns (small KL/r) do not buckle* and simply *fail by material yielding*. Here steel reaches the value of fy or even fu.



2- *Long columns (large KL/r) usually fail by elastic buckling* mentioned above. Euler's equations are usually used to estimate the stresses for this type.

3 - Between short and long regions, the failure of the column occurs through *inelastic buckling*.





Types of compression members.

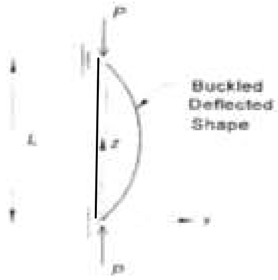
Square and rectangular tubing (g) and (h) are being used more each year. For many years, only a few steel mills manufactured steel tubing for structural purposes. Perhaps the major reason tubing was not used to a great extent is the difficulty of making connections with rivets or bolts. This problem has been fairly well eliminated, however, by the advent of modern welding. The use of tubing for structural purposes by architects and engineers in the years to come will probably be greatly increased for several reasons:

1. The most efficient compression member is one that has a constant radius of gyration about its centroid, a property available in round HSS tubing and pipe sections. Square tubing is the next-most-efficient compression member.
2. Four-sided and round sections are much easier to paint than are the six-sided open W, S, and M sections. Furthermore, the rounded corners make it easier to apply paint or other coatings uniformly around the sections.
3. They have less surface area to paint or fireproof.
4. They have excellent torsional resistance.
5. The surfaces of tubing are quite attractive.
6. When exposed, the round sections have wind resistance of only about two-thirds of that of flat surfaces of the same width.
7. If cleanliness is important, hollow structural tubing is ideal, as it doesn't have the problem of dirt collecting between the flanges of open structural shapes.

A slight disadvantage that comes into play in certain cases is that the ends of tube and pipe sections that are subject to corrosive atmospheres may have to be sealed to protect their inaccessible inside surfaces from corrosion. Although making very attractive exposed members for beams, these sections are at a definite weight disadvantage compared with W sections, which have so much larger resisting moments for the same weights.

3 - Euler Buckling of Columns

Buckling can be elastic (longer thin members) or inelastic (shorter members). Here we shall derive the Euler buckling (critical) load for an elastic column. Consider a long and slender compression member (hinged) as shown in the figure above. The Euler buckling formula is derived for an ideal or perfect case, where it is assumed that the column is long, slender, straight, homogeneous, elastic, and is subjected to concentric axial compressive loads. The differential equation for the lateral displacement v is given as:



EI y'' = - M
EI y'' = - py

Y'' + (p/EI) y = 0 let (p/EI) = w^2 …(1)

sub in above eq. we get

Y'' + w^2 y = 0 then y = A cos w + B sin w

Apply B.C. : at x = 0 , y = 0 the y = B sin w

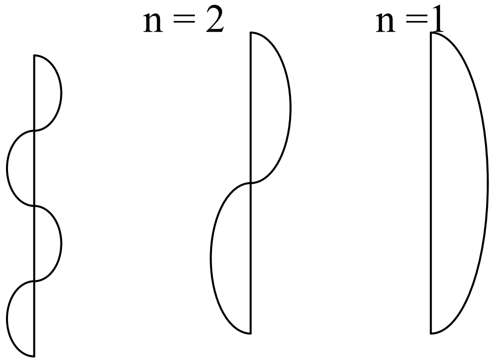
And at x = L y = 0   0 = B sin w L   but B ≠ 0

Then sin w L = 0 , $\sin^{-1} 0 = wL$, then w = (nπ /L)
n = 1,2 ,3 , …..

sub in eq ---(1) we get :

(p/EI) = ($n^2 \pi^2 / L^2$) then $P_{crit.} = n^2 \pi^2 EI / L^2$

Called **Euler** load or Euler buckling load



4 – Buckling Stress

$P_{cr} = n^2 \pi^2 EI / L^2$, $P_{cr} = \pi^2 EI / (L/n)^2$

Let $k = 1/n$, and since $I = r^2 A$

$P_{cr} = \pi^2 E r^2 A / (kL)^2$ k = effective length factor
L = effective length

$P_{cr} = \pi^2 E A / (kL/r)^2$, $P_{cr} / A = \pi^2 E / (kL/r)^2$

kL/r = slenderness ratio

$F_{cr} = \pi^2 E / (kL/r)^2$ is the ***critical buckling stress***

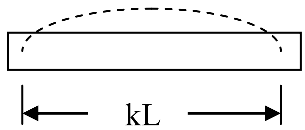
The student should carefully note that the buckling load determined from the Euler equation is independent of the strength of the steel used.

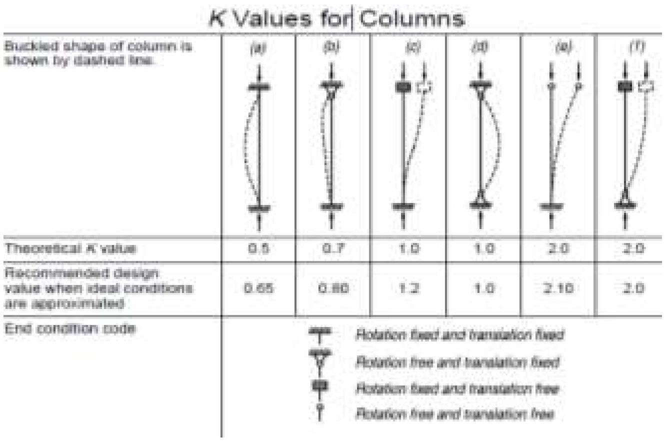
5 – End Restraints and Effective length of Columns:

The effective length is governed by column end restraints effective length (kL) which is the length between the inflection points of the column.

$(kL/r) \leq 200$
we get k from manual

Table C – C2.2 P. 240
AISC – M LRFD (previous versions)





K Values for Columns

Buckled shape of column is shown by dashed line.	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	Rotation fixed and translation fixed; Rotation free and translation fixed; Rotation fixed and translation free; Rotation free and translation free					

The selection of Fcr depends on the value of kL/r

6 – Design and Analysis of compression members (in the elastic range):

1 . Compute the slenderness ratio kL/r
2. Using the largest kL/r , obtain the design compressive strength Φc Fcr from table (4 -1 page 4 – 22 to 4 – 320).
3 – Compute the design compressive strength

Φc Pn = Φc Fcr Ag.

Φc = 0.9 for compression members design

Ex1: compute the design compressive strength for a W12*120 column that has an un-braced length of 16 ft and has a pin – connected ends. Use A36 steel.

Solution:

W shapes specifications rx = 5.51 and ry = 3.13 , Ag = 35.3 in^2

kL/r = 1*16*12/ 3.13 = 61 < 200 O.K

Φc Fcr = 26.6 ksi

Φc Pn = Φc Fcr Ag. , Φc Pn = 26.6 * 35.3 = 938.98 kips

Notes :

The *nominal compressive strength*, P_n, shall be determined based on the *limit state* of *flexural buckling*.

$$P_n = F_{cr} A_g \quad \text{(E3-1)}$$

The *flexural buckling stress*, F_{cr}, is determined as follows:

(a) When $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$ (or $F_e \ge 0.44F_y$)

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \quad \text{(E3-2)}$$

(b) When $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$ (or $F_e < 0.44F_y$)

$$F_{cr} = 0.877 F_e \quad \text{(E3-3)}$$

where

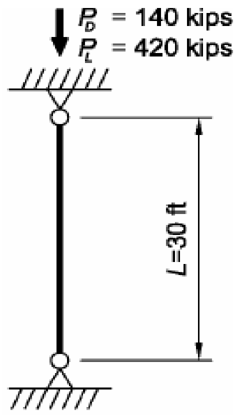
F_e = elastic critical buckling stress determined according to Equation E3-4, Section E4, or the provisions of Section C2, as applicable, ksi (MPa)

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad \text{(E3-4)}$$

User Note: The two equations for calculating the limits and applicability of Sections E3(a) and E3(b), one based on KL/r and one based on F_e, provide the same result.

Ex2: Given W-shape column designed with pinned ends, select an ASTM A992 (fy = 50 ksi) W-shape column to carry an axial dead load of 140 kips and live load of 420 kips. The column is 30 ft long, and is pinned top and bottom in both axes. Limit the column size to a nominal 14 in. shape.

Solution:



Pu = 1.2(140 kips) + 1.6(420 kips) = 840 kips

For a pinned-pinned condition, K = 1.0

Assume KL/r = 100

Φc Fcr = 21.7 ksi

Ag = Pu/ Φc Fcr = 840/21.7 = 38.7 in^2

Then from W tables select W14*132 (Ag = 38.8 in^2 , rx = 6.28 , ry = 3.76 in)

$$\frac{KL}{r_x} = \frac{1.0(30\,ft)(12\,in/ft)}{6.28\,in} = 57.32$$

$$\frac{KL}{r_y} = \frac{1.0(30\,ft)(12\,in/ft)}{3.76\,in} = 95.74 \text{ use larger}$$

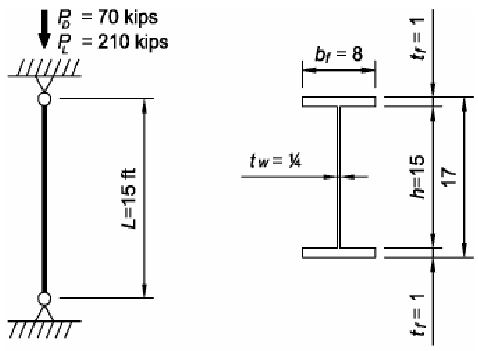
Φc Fcr = 23.02 ksi (By Interpolation)

Φc Pn = Φc Fcr Ag = (23.02) (38.8) = 893 ksi > 840 O.K.

Its important to check Pu/ Φc Pn ≥ 0.9

Pu / Φc Pn = 893/840 = 0.94 V. good

Ex3: Is the built-up, ASTM A572 grade 50, column with PL1in.×8in. flanges and a PL4in.×15in. web sufficient to carry a dead load of 70 kips and live load of 210 kips in axial compression ? The column length is 15 ft and the ends are pinned in both axes.



Solution:

ASTM A572 Grade 50 Fy = 50 ksi Fu = 65 ksi

Built-up Column d = 17.0 in. b_f = 8.00 in. t_f = 1.00 in. h = 15.0 in. t_w = 0.250 in.

Calculate the required strength:

Pu = 1.2(70.0 kips) + 1.6(210 kips) = 420 kips

A = 2(8.00 in.)(1.00 in.) + (15.0 in.)(0.250 in.) = 19.8 in^2

$$I_y = \frac{2(1.00\text{ in.})(8.00\text{ in.})^3}{12} + \frac{(15.0\text{ in.})(0.250\text{ in.})^3}{12} = 85.4\text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{85.4\text{ in.}^4}{19.8\text{ in.}^2}} = 2.08\text{ in.}$$

$$I_x = \sum Ad^2 + \sum I_x$$

$$= 2(8.00\text{ in.}^2)(8.00\text{ in.})^2 + \frac{(0.250\text{ in.})(15.00\text{ in.})^3}{12} + \frac{2(8.0\text{ in.})(1.0\text{ in.})^3}{12} = 1100\text{ in.}^4$$

For a pinned-pinned condition, K = 1.

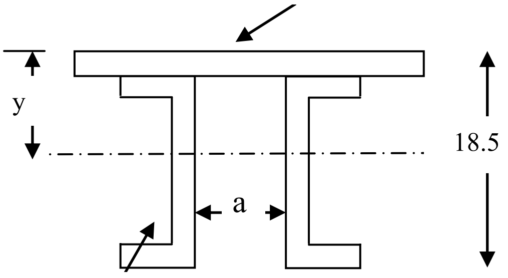
Since the unbraced length is the same for both axes, the y-y axis will govern by inspection.

$$\frac{KL_y}{r_y} = \frac{1.0(15.0\text{ ft})}{2.08\text{ in.}}\frac{12.0\text{ in.}}{\text{ft}} = 86.6$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2(29{,}000\text{ ksi})}{(86.6)^2} = 38.2\text{ ksi}$$

Φc Pn = Φc Fcr Ag = (25.65) (19.8) = 507 ksi > 420 O.K.

Ex4 : For the built up section shown in Fig. below find the max. allowable load P if kL = 19 ft. Use A36. a = 12".



Solution:
For MC 18*42.7
A = 12.6 in^2
d = 18"
Ix = 554 in^4
Iy = 14.4 in^4
tw = 0.45"
tf = 0.625 "
A = 2*12.6 + 20*0.5 = 35.2 in^2

$\bar{y}_{\text{from top}}$ = (10*0.25 + 2*12.6 *9.5)/ 35.2 = 6.87 in.

$Ix = 2*554 + 2* 12.6* (9.5 - 6.87)^2 + 20*(0.5)^3 /12 + 10*(6.62)^2 = 1721\ in^4$
$Iy = 2*14.4 + 12.6 *2 (6.877)^2 + 1/12 *(20)^3 *0.5 = 1554\ in^4$

ry = 6.64 in
(kl /r)y = 19*12/ 6.64 = 34.34 then Φc Fcr = 20.27 ksi

Φc Pn = Φc Fcr Ag = 0.9 *20.27 * 35.2 = **642.15 kips**