Experiment No.13

The Series RLC Resonance Circuit

Object

To perform be familiar with The Series RLC Resonance Circuit and their laws.

Theory

Thus far we have studied a circuit involving a (1) series resistor R and capacitor C circuit as well as a (2) series resistor R and inductor L circuit. In both cases, it was simpler for the actual experiment to replace the battery and switch with a signal generator producing a square wave. The current through and voltage across the resistor and capacitor, and inductor in the circuit were calculated and measured.

This lab involves a resistor R, capacitor C, and inductor L all in series with a signal generator and this time is experimentally simpler to use a sine wave that a square wave. Also, we will introduce the generalized resistance to AC signals called "impedance" for capacitors and inductors. The mathematical techniques will use simple properties of complex numbers which have real and imaginary parts. This will allow you to avoid solving differential equations resulting from the Kirchhoff loop rule and instead you will be able to solve problems using a generalized Ohm's law. This is a significant improvement since Ohm's law is an algebraic equation which is much easier to solve than differential equation. Also, we will find a new phenomenon called "resonance" in the series RLC circuit.

Consider now the driven series RLC circuit shown in Figure 1.

Applying Kirchhoff's loop rule, we obtain

$$V(t)-V_R(t)-V_L(t)-V_C(t)=V(t)-IR-L\frac{dI}{dt}-\frac{Q}{C}=0$$
.......(1)

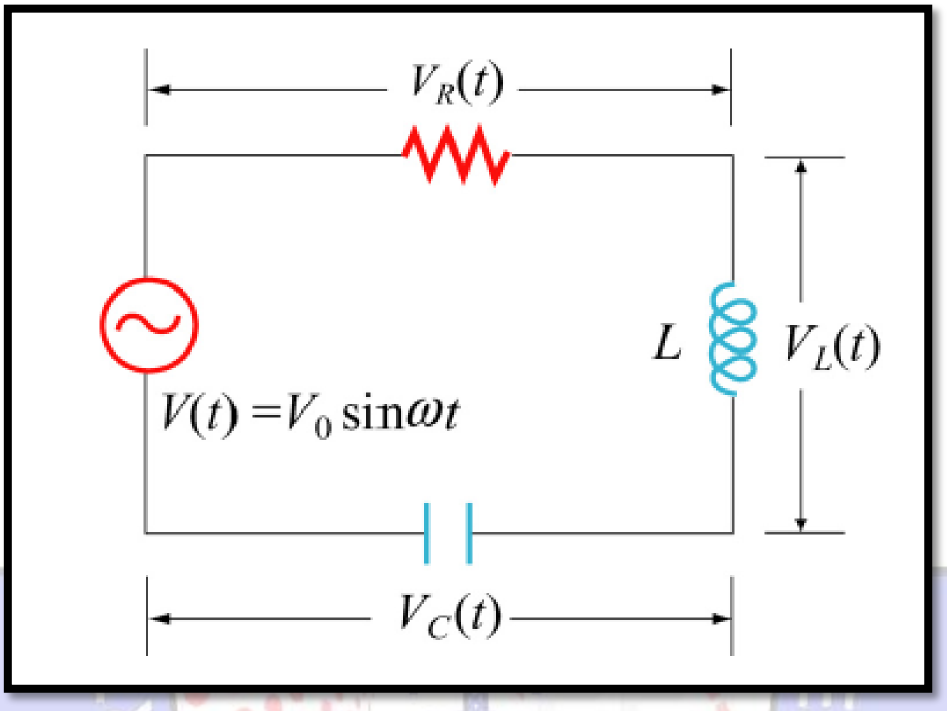

Figure 12.3.1 Driven series RLC Circuit

which leads to the following differential equation:

$$L\frac{dI}{dt}+IR+\frac{Q}{C}=V_0\sin\omega t$$
..............................(2)

Assuming that the capacitor is initially uncharged so that $I=+dQ/dt$is proportional to the increase of charge in the capacitor, the above equation can be rewritten as

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=V_0\sin\omega t$$
..........................(3)

One possible solution to Eq. (3) is

$$Q(t) = Q_0 \cos(\omega t - \phi)$$ ....(4)

where the amplitude and the phase are, respectively,

$$Q_0 = \frac{V_0 / L}{\sqrt{(R\omega / L)^2 + (\omega^2 - 1/LC)^2}} = \frac{V_0}{\omega\sqrt{R^2 + (\omega L - 1/\omega C)^2}} = \frac{V_0}{\omega\sqrt{R^2 + (X_L - X_C)^2}}$$ .....(5)

And

$$\tan\phi = \frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right) = \frac{X_L - X_C}{R}$$ ....(6)

The corresponding current is

$$I(t) = +\frac{dQ}{dt} = I_0 \sin(\omega t - \phi)$$ ....(7)

with an amplitude

$$I_0 = -Q_0\omega = -\frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$ ....(8)

Notice that the current has the same amplitude and phase at all points in the series RLC circuit. On the other hand, the instantaneous voltage across each of the three circuit elements R, L and C has a different amplitude and phase relationship with the current, as can be seen from the phasor diagrams shown in Figure 12.3.2.

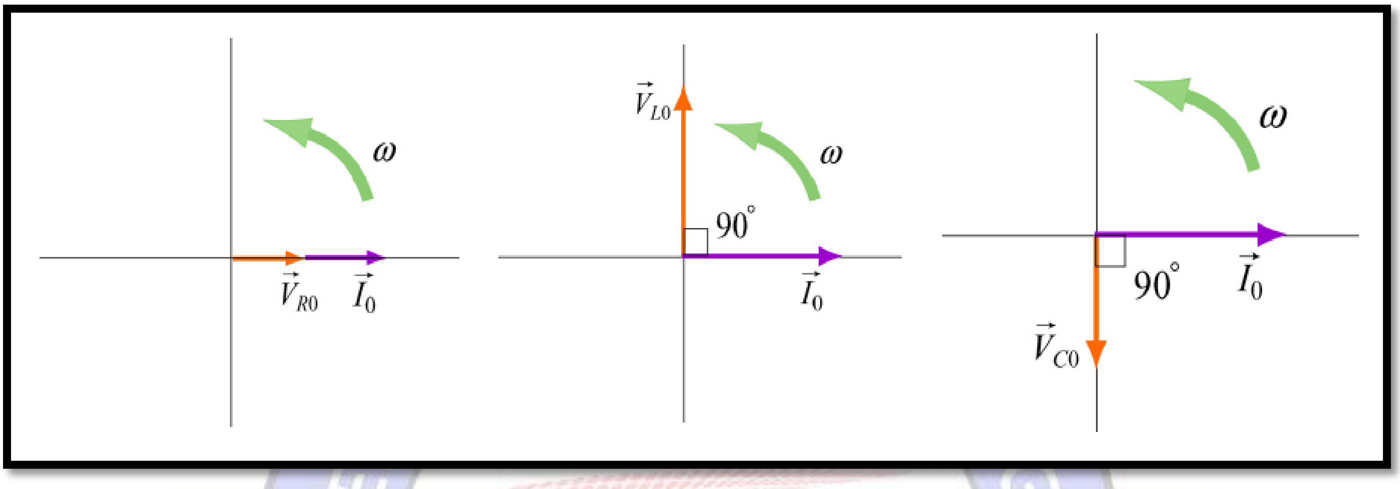

Figure .2 Phasor diagrams for the relationships between current and voltage in (a) the resistor, (b) the inductor, and (c) the capacitor, of a series RLC circuit.

From Figure .2, the instantaneous voltages can be obtained as:

$$V_R(t) = I_0 R \sin \omega t = V_{R0} \sin \omega t$$
$$V_L(t) = I_0 X_L \sin\left(\omega t + \frac{\pi}{2}\right) = V_{L0} \cos \omega t$$
$$V_C(t) = I_0 X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -V_{C0} \cos \omega t$$

.....................(9)

where

$$V_{R0} = I_0 R, \quad V_{L0} = I_0 X_L, \quad V_{C0} = I_0 X_C$$

.....................(10)

are the amplitudes of the voltages across the circuit elements. The sum of all three voltages is equal to the instantaneous voltage supplied by the AC source:

$$V(t)=V_R(t)+V_L(t)+V_C(t)$$ ....(11)

Using the phasor representation, the above expression can also be written as

$$\vec{V}_0=\vec{V}_{R0}+\vec{V}_{L0}+\vec{V}_{C0}$$ ....(12)

as shown in Figure 3 (a). Again, we see that current phasor $\vec{I_0}$ leads the capacitive voltage phasor by $\overrightarrow{V_{CO}}$ by $\frac{\pi}{2}$ but lags the inductive voltage phasor $\overrightarrow{V_{LO}}$ by $\frac{\pi}{2}$. The three voltage phasors rotate counterclockwise as time passes, with their relative positions fixed.

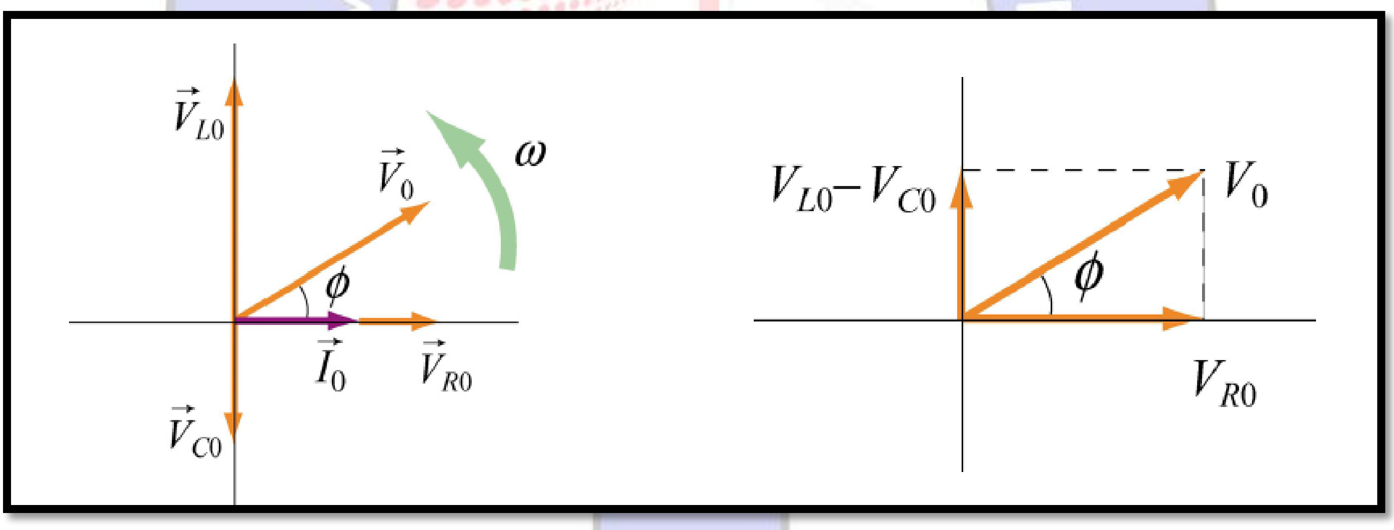

Figure .3 (a) Phasor diagram for the series RLC circuit. (b) voltage relationship

The relationship between different voltage amplitudes is depicted in Figure 3(b). From the Figure, we see that

$$
\begin{aligned}
V_0 &= |\vec{V}_0| = |\vec{V}_{R0} + \vec{V}_{L0} + \vec{V}_{C0}| = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} \\
&= \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} \\
&= I_0 \sqrt{R^2 + (X_L - X_C)^2}
\end{aligned}
$$

...........(13)

which leads to the same expression for I0 as that obtained in Eq. (7).

It is crucial to note that the maximum amplitude of the AC voltage source V_0 is not equal to the sum of the maximum voltage amplitudes across the three circuit elements:

$$V_0 \neq V_{R0} + V_{L0} + V_{C0}$$

......................................(14)

This is due to the fact that the voltages are not in phase with one another, and they reach their maxima at different times.

We have already seen that the inductive reactance $X_L=\omega L$ and capacitance reactance $X_C=1/\omega C$ play the role of an effective resistance in the purely inductive and capacitive circuits, respectively. In the series RLC circuit, the effective resistance is the impedance, defined as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

...............................(15)

The relationship between Z, X_L and X_C can be represented by the diagram shown in Figure .4:

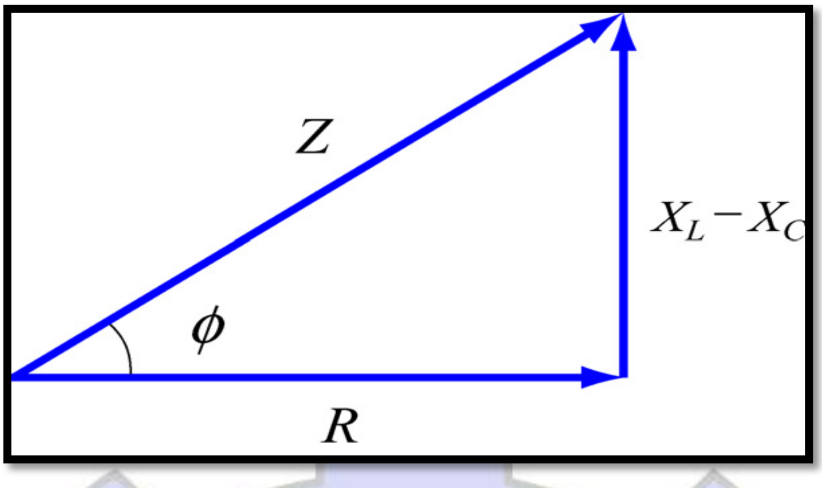

Figure .4 Diagrammatic representation of the relationship between Z, XL and XC

The impedance also has SI units of ohms. In terms of Z, the current may be rewritten as

$$I(t) = \frac{V_0}{Z}\sin(\omega t - \phi) \quad \text{.................................(16)}$$

Notice that the impedance Z also depends on the angular frequency ω, as do XL and XC

Using Eq. (6) for the phase φ and Eq. (15) for Z, we may readily recover the limits for simple circuit (with only one element). A summary is provided in Table.1 below:

Simple Circuit	R	L	C	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$	$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
purely resistive	R	0	∞	0	0	0	R
purely inductive	0	L	∞	X_L	0	$\pi/2$	X_L
purely capacitive	0	0	C	0	X_C	$-\pi/2$	X_C

Table 1 Simple-circuit limits of the series RLC circuit

Resonance

Eq. (15) indicates that the amplitude of the current $I_0 = V_0/Z$ reaches a maximum when Z is at a minimum. This occurs when $X_L = X_C$, or ωL=1/ωC , leading to

$$\omega_0 = \frac{1}{\sqrt{LC}}$$ ....(17)

The phenomenon at which I_0 reaches a maximum is called a resonance, and the frequency ω_0 is called the resonant frequency. At resonance, the impedance becomes Z=R, the amplitude of the current is

$$I_0 = \frac{V_0}{R}$$ ....(18)

and the phase is

$$\phi = 0$$ ....(19)

as can be seen from Eq. (5). The qualitative behavior is illustrated in Figure 5.

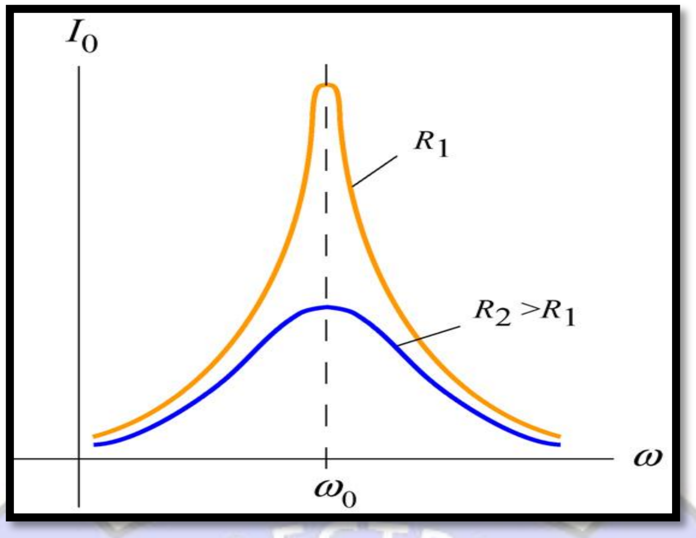

Figure 5 The amplitude of the current as a function of ω in the driven RLC circuit.

Procedure

1. Connect the resistance, capacitance and inductance in series as shown in Fig.6

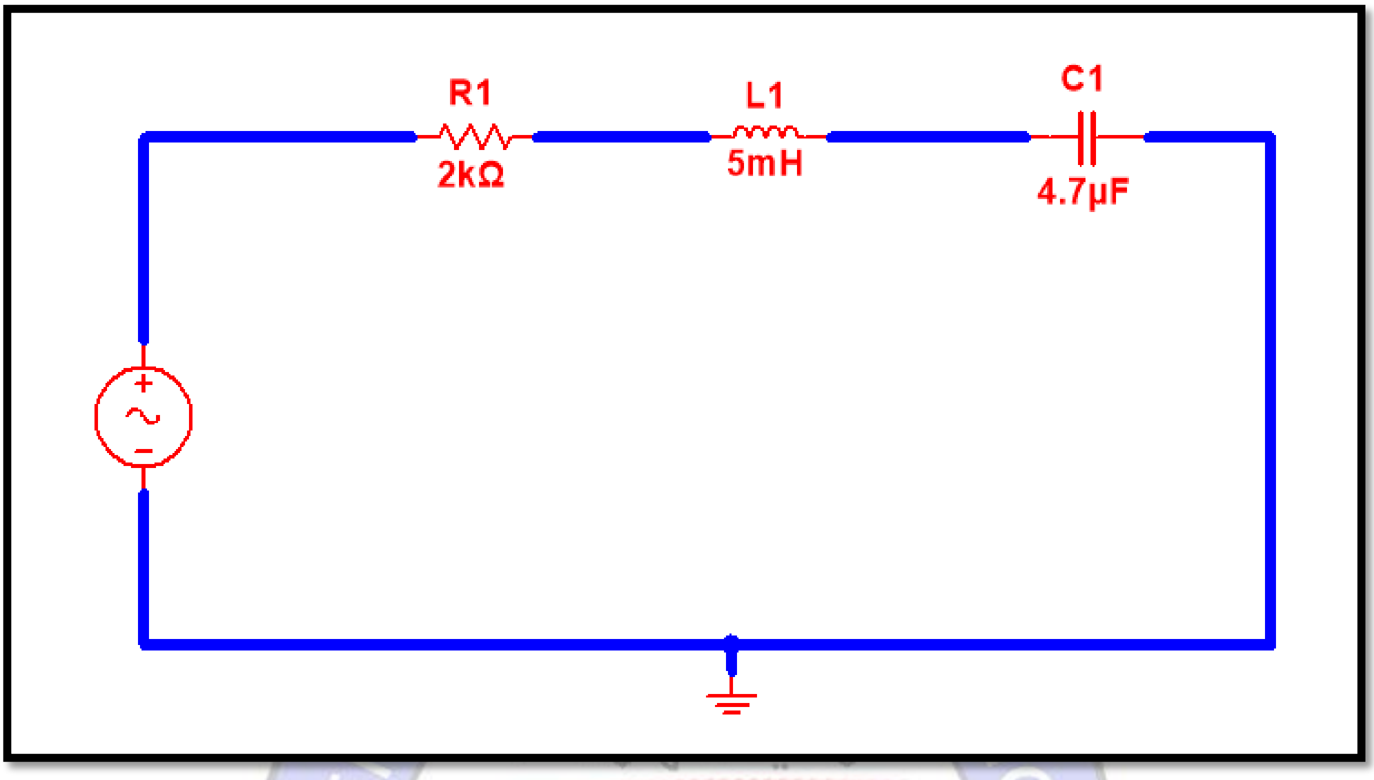

Fig.6

2. Adjust the function generator to 200 Hz, 10 V and 50% duty cycle.
3. Select sine, waveform, turn the circuit ON for about 50 ms then turn it OFF.
4. Measure the phase shift between the current I and the input voltage V using oscilloscope.
5. Draw the result of the display graph.
6. Draw the Phasor diagram.

Resonance

1. Connect the circuit as shown in Fig. (7).
2. Set the voltmeter to 6 Vrms.
3. Select sine waveform; vary the oscillator from 14 kHz to 17 KHz in steps of 0.5 kHz.
4. Record the reading of the voltmeter at each step as in table (2)

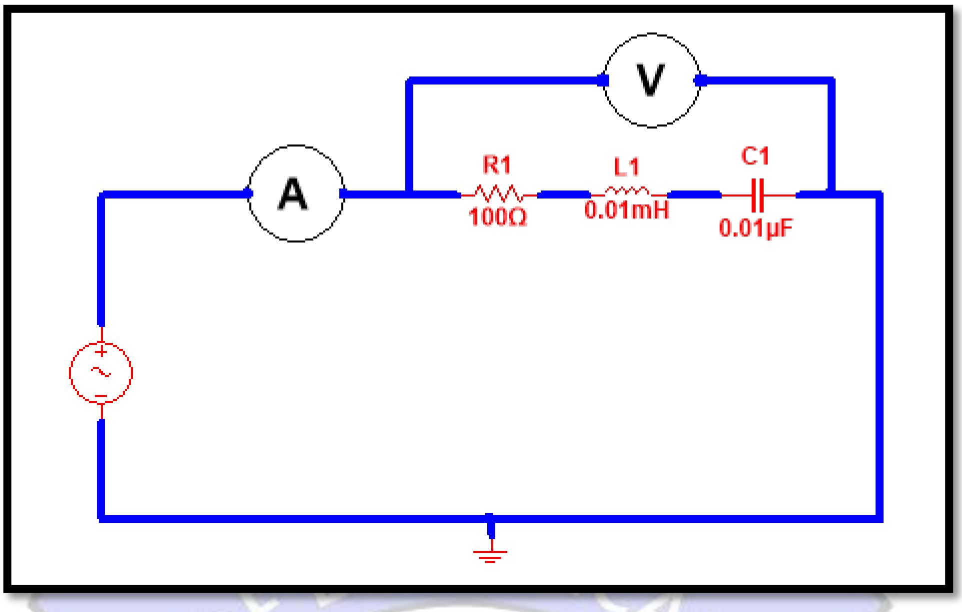

Fig. (7).

Frequency(kHz)	$I_{r.m.s}$(A)

table (2)

5. Evaluate the impedance of the circuit at each step.
6. Plot a graph of X_L, and X_C w.r.t. of frequency
7. Plot a graph of impedance w.r.t. of frequency
8. Determine the value of the impedance at the resonant frequoncy.
9. Compare the value of the resonant frequency to the theoretical value.

Discussion

1. Can we obtain a plot of X_L, against frequency f experimentally?
2. Explain why Phasor and impedance diagrams have the same angles.
3. What is the value of Phasor shift if R= 300Ω L= 400mH with f=50Hz. Discuss the increase or decrease in the phase shift.
4. Comment on the result you have obtained